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A Novel Classification based on Ellipse for Face Recognition

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Abstract. *This paper, we presented a novel classification based on the nearest and the center of ellipse (NCE) for face recognition. Any ellipse is generated by three points, which are obtained from the combination of m distinct points taken from the same class: m is a number of image per class. The classification criteria are as follows: 1) if the test image is inside or is on the ellipse, we calculate the distance between the test image and the center of the ellipse; 2) if the test image is outside the ellipse, we find the minimum distance. The distance between the test image and the z point such that the z point is any point in the three points in the ellipse then the test is in class C_{NCE} when the distance that is the shortest of all of classes. A large number of experiments were investigated on the Faces94 and the Grimace database. Meanwhile, the comparison between the NCE and the other neighborhood-based classification methods; including the nearest feature line, the shortest feature line segment, and the restricted nearest feature line with ellipse.*
Keywords: Face recognition, Classifier, Ellipse

1. **Introduction.** One of the most research problems of the image processing is a face recognition, which is always interesting because of its applications. For instance, Kirby and Sirovich [1] presented a specified framework for the representation of faces. Also, Cottrell and Fleming [2] proposed how to extract image features for pattern recognition automatically. Besides, Turk and Pentland [3] presented the principal component analysis, which is a face identification approach, and the other several methods [4,5,6,7,8]. Moreover, various pattern classification methods have been proposed until now; for example, Cover and Hart [9] proposed the nearest neighbor (NN) classifier, which is a simple non-parametric classification; in the meantime, Li and Lu [10] improved the nearest neighbor, which is called the nearest feature line (NFL). Furthermore, Zhou, Zhang and Wang [11] extended the nearest feature line, which calculates the product of the distance between the test and the two points in a training set. Likewise, Han, Han and Yang [12] investigated the shortest feature line segment (SFLS): the SFLS used circle and tried to find the shortest feature line segment. The restricted nearest feature line with ellipse (RNFL) is proposed by Feng, Pan, and Yan [13], the RNFL improved the miss classification of NFL and it used the ellipse to restrict the feature line.

In this paper, we propose a novel modified the NFL and the RNFL classification approach called the nearest and center of ellipse (NCE). Instead of calculating the distance between the test image and the projection of the test image, the NCE attempts to find the distance between the test image and the center of the ellipse. The classification criteria are as follow:

1) if the test image is inside or is on the ellipse, we calculate the distance between the center of the ellipse and the test image;

2) if the test image is outside the ellipse, we find the minimum distance. The distance between the test image and the z point such that the z point is any point in the three points in the ellipse then the test is in class C_{NCE} when the distance that is the shortest of all of classes. The comparison was performed by the nearest feature line (NFL), the shortest feature line segment (SFLS), and the restricted nearest feature line with ellipse (RNFLE).

2. **Background.** In this section, we introduce the nearest feature line, the shortest feature line segment, and the restricted nearest feature line with the ellipse. Suppose z_i^c, z_j^c are the two points the training set from the same class $c = 1, 2, 3, \ldots, N$: N is the number of class and let z be the test image. For the convenience, define $d(u, v)$ as the Euclidean distance between vectors u and v.

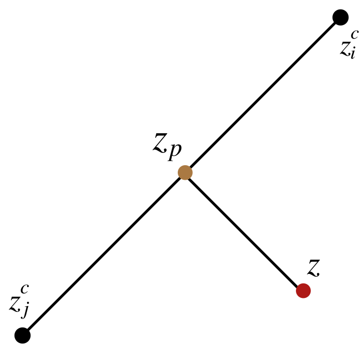

FIGURE 1. The nearest feature line

2.1. **The nearest feature line (NFL).** $\overline{z_i^c z_j^c}$ represents the line segment which is passing through z_i^c and z_j^c called a feature line (FL) of the class C_{NFL}, as shown in figure 1. Define z_p as the projection point of z which can be calculated by

$$z_p = z_i^c + t(z_j^c - z_i^c), \tag{1}$$

where $t = \frac{(z_p - z_i^c)^T (z_j^c - z_i^c)}{(z_j^c - z_i^c)^T (z_j^c - z_i^c)}$. The distance between the test image z and the feature line $\overline{z_i^c z_j^c}$ can be calculated by

$$D_1(z, \overline{z_i^c z_j^c}) = d(z, z_p). \tag{2}$$

The classification decision called the nearest feature distance can be defined as follow:

$$C_{\text{NFL}} = \arg\min_c \left\{ \min_{1 \le i < j \le m} D_1(z, \overline{z_i^c z_j^c}) |\ c = 1, 2, 3, \ldots, N \right\}, \tag{3}$$

where C_{NFL} is the class of the test image.

Moreover, the nearest feature line (NFL) may fail when the the test image in NFL are far away from the training image vector, which is called as extrapolation inaccuracy of NFL as shown in figure 2.

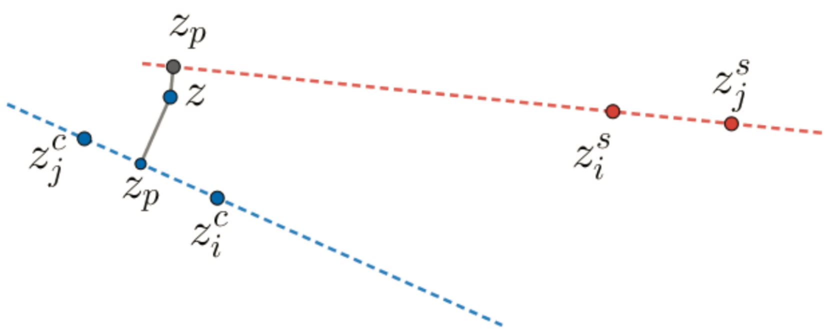

FIGURE 2. The extrapolation mistake of the nearest feature line

2.2. **The shortest feature line segment (SFLS).** This method finds the shortest feature line segment which is classified by the test, which is inside a circle of class as shown in figure 3. The creation of the circle form two training, the distance metric of SFLS can be calculated by

$$D_2(z, z_i^c z_j^c) = d(z_i^c, z_j^c). \tag{4}$$

The classification decision can be defined as follow:

$$C_{\text{SFLS}} = \arg\min_c \left\{ \min_{1 \le i < j \le m} D_2(z, z_i^c z_j^c) | c = 1, 2, 3, \ldots, N \right\}, \tag{5}$$

where C_{SFLS} is the class of the test image.

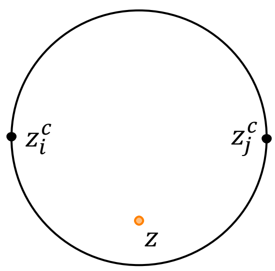

FIGURE 3. The metric of the shortest feature line segment

2.3. **The restricted nearest feature line with ellipse (RNFLE).** The main idea of this method, which uses ellipse to restrict the feature line. Define z_i^c, z_j^c as foci of any ellipse like figure 4, and a_0 as the ratio between the length of ellipse major axis and the length of the center to either focus.

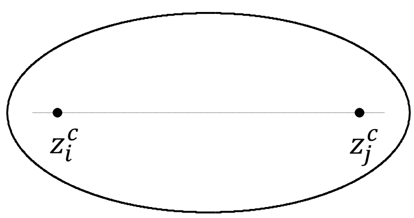

FIGURE 4. z_i^c, z_j^c are foci of the ellipse

The classification decision can be defined as follows:
1. If the test image (z) is inside the ellipse ($||z - z_i^c|| + ||z - z_j^c|| \leq a_0||z_i^c - z_j^c||$) which is shown in figure 5 (Left), the distance between the test image and the feature line $\overline{z_i^c z_j^c}$ is as follow:

$$D_3(z, \overline{z_i^c z_j^c}) = d(z, z_{ijp}^c), \tag{6}$$

where z_{ijp}^c represents the projection of z on the feature line $\overline{z_i^c z_j^c}$.
2. If the test image (z) is outside the ellipse ($||z - z_i^c|| + ||z - z_j^c|| > a_0||z_i^c - z_j^c||$) which is shown in figure 5 (Right), the distance between the test image and the feature line is as follow:

$$D_3(z, \overline{z_i^c z_j^c}) = \min\{d(z, z_i^c), d(z, z_j^c)\}. \tag{7}$$

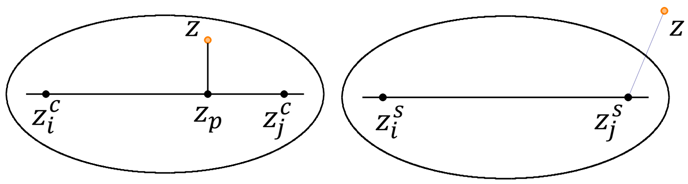

FIGURE 5. The left figure shows the test image is inside the ellipse and the right figure shows the test image is outside the ellipse

The test image is classified into class C_{RNFLE},

$$C_{\text{RNFLE}} = \arg\min_c \left\{ \min_{1 \leq i < j \leq m} D_3(z, \overline{z_i^c z_j^c}) | c = 1, 2, 3, \ldots, N \right\}. \tag{8}$$

3. **The proposed method.** Let N be the number of the class, m be the number of images per class and $n = mN$. We use the principal component analysis (PCA) to find a subset of the principle component in a set of training 2 Dimensional faces; then we project faces into the principal components space which can be gathered the feature vectors (z_i). The process of principal component analysis is described as follows
Step 1. Set x_i be the image vector of image and calculate the mean of face images is defined by

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i. \tag{9}$$

Step 2. Calculate the covariance matrix of training images matrix by $C = \frac{1}{n}AA^T$, where

$$A = [(x_1 - \bar{x}) \ \ (x_2 - \bar{x}) \ \ (x_3 - \bar{x}) \ \ \ldots \ \ (x_n - \bar{x})]. \tag{10}$$

Step 3. Since the matrix C is high dimension, the eigenvectors of are considered by the matrix $L = \frac{1}{n}A^T A$ of size $n \times n$ (if λ is eigenvalue of L, then λ is also eigenvalue of C). Let V_i be the eigenvector of matrix L corresponding to the eigenvalue λ_i where $|\lambda_1| \geq |\lambda_2| \geq \cdots |\lambda_{n_1}|$. Thus, $U_i = AV_i$ is eigenvector of C corresponding to the eigenvalue λ. The eigenface is defined by

$$U = [U_1 \ \ U_2 \ \ U_3 \ \ \ldots \ \ U_n]. \tag{11}$$

Step 4. (Return to Vector Conversion) The weight of each eigenvector represents the image in the eigenface space as given by

$$z_i = U^T(x_i - \bar{x}), \tag{12}$$

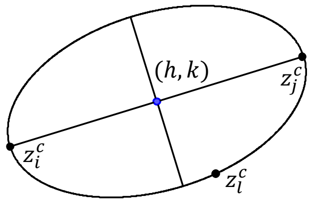

FIGURE 6. Generalizing an ellipse from 3 points

where U is the eigenface.
For the convenience, define $\text{diam}(A) = \max\{d(u,v)|u,v \in A\}$.

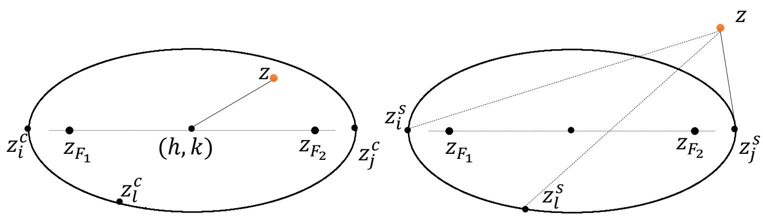

FIGURE 7. The left figure shows the test image is inside the ellipse and the right figure shows the test is outside the ellipse

The nearest and the center of ellipse (NCE)
Suppose z_i^c, z_j^c and z_l^c are the feature vectors from the PCA, which also are the three points training set from the same class c, $1 \le i < j < l \le m, 1 \le c \le N$. For this method, each class needs at least three points. The ellipse is generalized by the three points, which are obtained from the combinations of m distinct points taken of the same class, that is the ellipse is created from z_i^c, z_j^c and z_l^c. Without loss of generality, setting $d(z_i^c, z_j^c) = \text{diam}(\{z_i^c, z_j^c, z_l^c\})$. The center (h,k) is the midpoint of the z_i^c and z_j^c while $a_{c_{ijl}} = \frac{1}{2}d(z_i^c, z_j^c)$ and

$$b_{c_{ijl}} = \left\| \sqrt{\frac{(-(x_l-h)\sin(\theta)+(y_l-k)\cos(\theta))^2}{1-\frac{((x_l-h)\cos(\theta)+(y_l-k)\sin(\theta))^2}{a^2_{c_{ijl}}}}} \right\|_2 \tag{13}$$

with $z_l = [x_l, y_l]$.

The foci of the ellipse can be calculated by

$$z_{F_1} = [h,k]^T - C[\cos\theta, \sin\theta]^T, \tag{14}$$

$$z_{F_2} = [h,k]^T + C[\cos\theta, \sin\theta]^T, \tag{15}$$

where $C = \sqrt{a^2 - b^2}$ and $\theta = \arctan\left(\frac{y_i - y_j}{x_i - x_j}\right)$. Define z as the test image and the sum of distances between the test image and the foci of the ellipse is denoted by

$$D_4(z, z_{ijl}^c) = d(z, z_{F_1}) + d(z, z_{F_2}). \tag{16}$$

Without loss of generality, let $a > b$. By definition of the ellipse, we acquire that the detailed procedure of judging is as follow: $D_4(z, z_{ijl}^c) \le 2a$ if as shown in figure 7 (Left), the test image (z) is inside or is on the ellipse. Or else, $D_4(z, z_{ijl}^c) > 2a$ as shown in figure 7 (Right), the test image is outside the ellipse.

The classification decision can be defined as follows:

1. If the test image (z) is inside the ellipse and $z_{c_{ijl}} = [h,k]^T$ represents the center of the

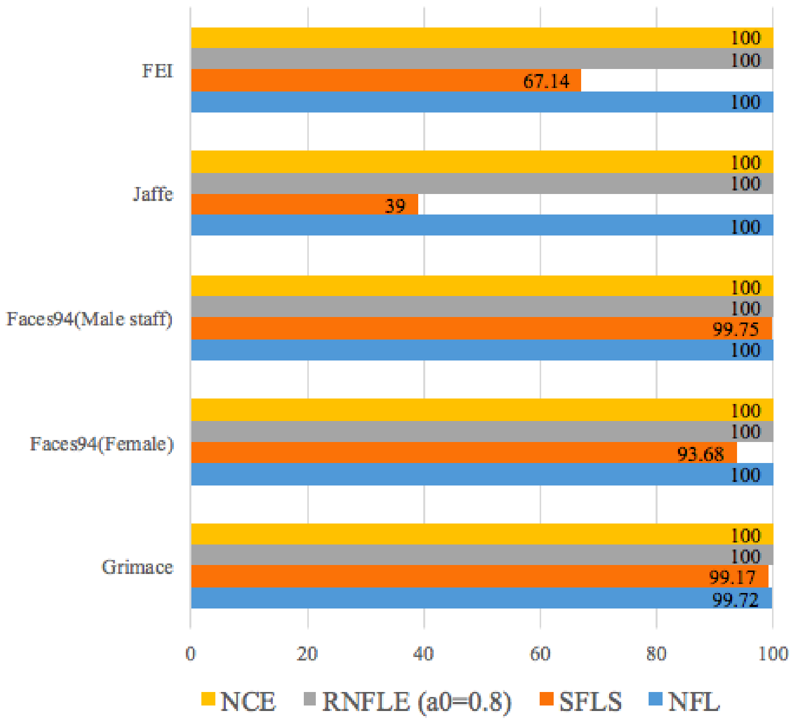

FIGURE 8. The recognition rate of the first experiment, the test image is in the training set.

ellipse, the distance between the test image and the center of the ellipse is as follow:

$$D_5(z, \overline{z_i^c z_j^c z_l^c}) = d(z, z_{c_{ijl}}). \tag{17}$$

2. If the test image (z) is outside the ellipse, or three points lie on the same line, or three points generate equilateral triangle, the distance between the test image and each point in three points of the ellipse is as follow:

$$D_5(z, \overline{z_i^c z_j^c z_l^c}) = \min\{d(z, z_i^c), d(z, z_j^c), d(z, z_l^c)\}. \tag{18}$$

The test image is classified into class C_{NCE},

$$C_{\text{NCE}} = \arg\min_c \left\{ \min_{1 \le i < j < l \le m} D_5(z, \overline{z_i^c z_j^c z_l^c}) | \quad c = 1, 2, 3, \ldots, N \right\}. \tag{19}$$

4. **Experimental results.** The criterions of the experiments are as follows: the nearest feature line (NFL), the shortest feature line segment (SFLS), and the restricted nearest feature line with ellipse (RNFLE). The databases are used by Grimace [14], FEI [15], JAFFE [16] and Faces94 (Female and Female Staff) [14] when define m as the number of image per class. Since FEI database have 200 people, we chose only 15 people for experimentation. Before we verify class of face image by proposed algorithm (NCE), we transform the training image and test image as column vector, then input these vectors to PCA process and verify by NCE.

Each class is divided into two sets, which are the test set and the training set when the test set includes the test image. We divided the experiment into three parts. Firstly, the training set is all images in the class, which includes m images per class as well as the test set; we chose a test image from the test set. Secondly, the training set did not include all images in the class, which consisted of $m - 1$ images per class while the remaining image was the test set; we chose the test image from the test set. Finally, the training set did

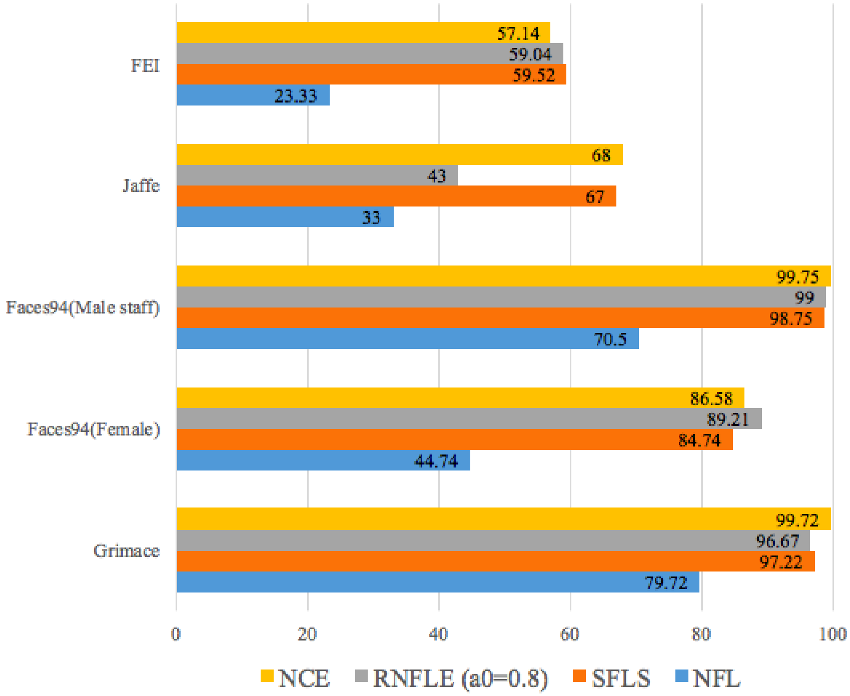

FIGURE 9. The recognition rate of the second experiment, the test image is not in training set and number of the training set= $m - 1$.

not all images in class, which consisted of $m - 2$ images per class and the remaining two images was the test set; we chose the test image from the test set. The recognition rate of three experiments, which is shown in figure8–10, respectively.

5. **Conclusions.** In this paper, the feature extraction of the face recognition based on principal components analysis. We presented a novel classification method which is called NCE. The measurement of the distance is the Euclidean distance. For the classification decision, if the test image is inside or is on the ellipse of class, we calculate the Euclidean distance of the center of the ellipse and the test; if the test image is outside the ellipse, we find the minimum distance, which that between the test and each point in the three points in the ellipse then the test is in class when the distance that is the shortest in all classes. The performance of NCE is compared using the NFL, SFLS, and RNFLE. Meanwhile, the experiment results on Grimace, FEI, Jaffe and Faces94 face database.

It is evident that the recognition rate of the RNFLE and the NCE are better than the SFLS and the NFL because the SFLS cannot classify in case of the data is very distributed. The NFL has the extrapolation mistake as figure 2. For the RNFLE algorithm, we must define a_0 (the ration between the length of ellipses long axis and the focus length of ellipse), it's wasted time for finding a suitable a_0, then we proposed the NCE that we do not necessary define a_0; besides, the NCE is better than the RNFLE. Only in the first experiment, the RNFLE and the NCE methods are more outperform than the NFL and the SFLS. Likewise, the NCE of Grimace, Jaffe and Faces94 (Male staff) database is better than the NFL, SFLS and RNFLE in the second experiment and the RNFLE of Faces94 (Female) database is better than the NFL, SFLS and NCE in the second experiment. Finally, the NCE which was used in all databases (accept FEI and Face94

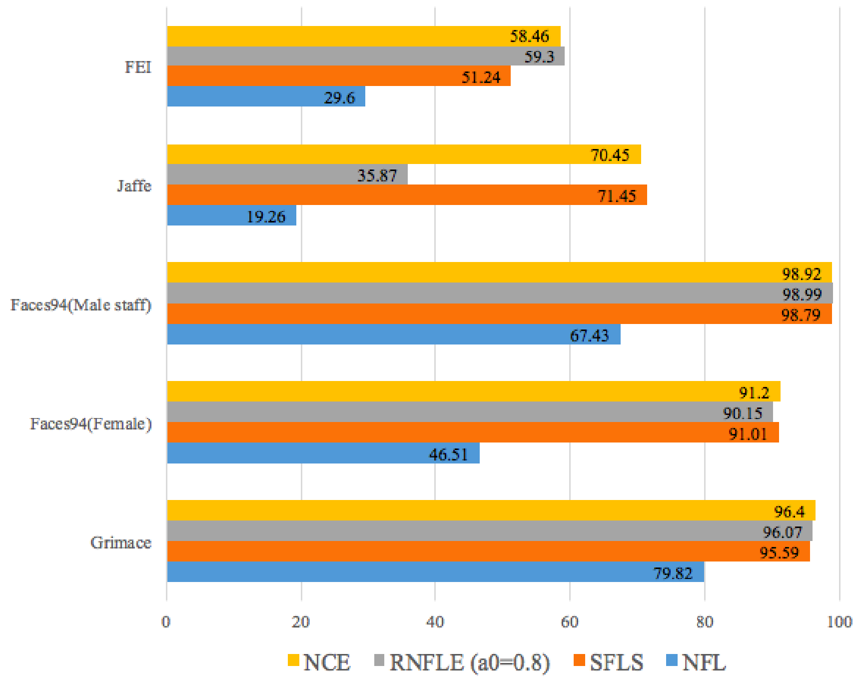

FIGURE 10. The recognition rate of the third experiment, the test image is not in training set and number of the training set= $m-2$.

(Male staff) database) is better than the NFL, SFLS and RNFLE and the RNFLE of Faces94 (Male staff) and FEI database is better than the NFL, SFLS and NCE in the third experiment. Mostly, the recognition rate of the NCE is better than the others; therefore, we recommend the NCE to classify the face recognition.

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