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Finite-Wordlength Analysis of ADC and Receiving Filter for OFDM Baseband Transceiver

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Abstract. *In this paper, the finite-wordlength analysis of ADC and receiving filter for OFDM baseband transceiver is presented based on the system performance indexes, the quantized noises and the related parameters including bit-error rate (BER), signal-to-noise ratio (SNR), signal-to-quantization noise ratio (SQNR), peak-to-average ratio (PAR), etc. Furthermore, in order to obtain a cost-effective hardware design of baseband transceiver, the restrictive SNR degradation is defined as the design criterion of finite-wordlength analysis. Finally, a simulation C model of OFDM transceiver is established to evaluate consistency with the finite-wordlength analysis.*
Keywords: OFDM, Finite-Wordlength Analysis, BER, SNR, SQNR

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1. **Introduction.** In view of a digital baseband transceiver, the analog-to-digital converter (ADC) is a crucial functional block converting the received continuous-time signal to discrete-time sample. The quality of the digitized sample is decisive to make the baseband signal processing algorithm precisely fulfill in receiver and thus to enhance the system performance. However, for OFDM system, two types of distortion, such as quantization noise and peak-to-average ratio (PAR), for ADC can downgrade the quality of received signal and further degrade the system performance. Consequently, we are going to take quantization noise and PAR into account to analyze the finite wordlength of ADC. On the other hand, the transmitting and the receiving filters are employed to do the spectral shaping and isolate the out of band signal and noise, respectively. Similarly, both the quantized coefficient and the PAR of receiving filter are the important issues to determine the finite wordlength of receiving filtering.

In general, the finite-wordlength optimization for a digital baseband transceiver design is according to extensive simulation results. However, this approach is not a practical design methodology to fulfill a reliable digital baseband transceiver. In this paper, the mathematical analysis of finite wordlength for ADC and receiving filter is presented based on the system performance indexes, the quantization noises and the related system parameters including bit-error-rate (BER), signal-to-noise ratio (SNR), signal-to-quantization-noise ratio (SQNR), PAR, etc. Besides, a simulation model of IEEE 802.11a [1] OFDM transceiver, as presented in [2] and illustrated in Fig. 1, is used to demonstrate the finite wordlength analysis and evaluation. The related system parameters of IEEE 802.11a are also shown in Table 1, where the maximum carrier frequency offset is assumed ±40 ppm, namely ±232.2 KHz, with carrier frequency of 5.805 GHz in upper U-NII band and the channelization (or signal) bandwidth (BW) of 20 MHz. The total used subcarriers are 52 composed of data and pilot subcarriers. An entire symbol length is 80, which consists of cyclic prefix (CP) and DFT symbol length.

The paper is organized as follows, the design criterion of finite wordlength analysis is described in Section 2. The finite wordlength of ADC is derived in Section 3. Next, the finite wordlength of receiving filter is described in Section 4. The numerical simulation results are shown in Section 5. Finally, the conclusions are given in Section 6.

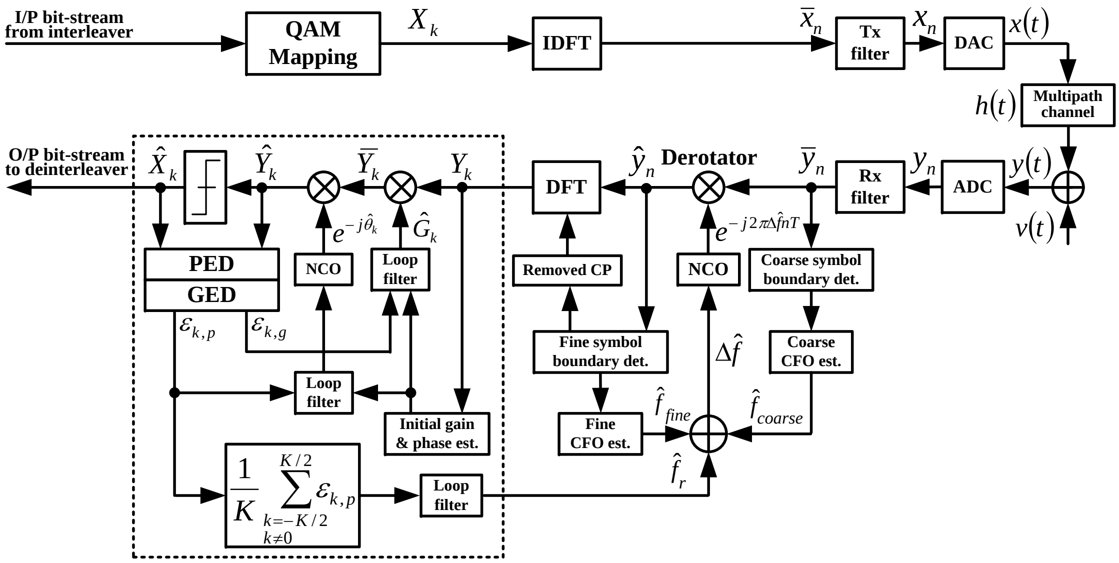


FIGURE 1. Block diagram of OFDM transceiver.

TABLE 1. OFDM system parameters.

Modulation	OFDM (BPSK, QPSK, 16-QAM, 64-QAM)
Maximum CFO	±40 ppm (±232.2 KHz)
Sampling frequency (f_s) (MHz)	40
Signal BW (f_B) (MHz)	20
IDFT/DFT point	64
Data (N_d)/pilot (N_p) subcarrier	48/4
Subcarrier spacing (f_Δ) (KHz)	312.5 (= f_B/N)
IDFT/DFT (T) period (μs)	3.2 μs
Symbol (N_s)/CP (N_g) length (sample)	80/16
Symbol (T_s)/CP (T_g) duration (μs)	4/0.8

2. Design Criterion of Finite Wordlength Analysis.

2.1. Quantization Noise. The quantization noise is generated while a infinite wordlength of physical signal is quantized and truncated to a finite wordlength signal such as ADC. The quantization process is regarded as a nonlinear system and thus defined as $\hat{x}_n =$

$Q(x_n)$, where x_n and $\hat{x}_n$ express the unquantized and the quantized signals of the nth sample, respectively. $Q(\cdot)$ is a nonlinear quantization process. In theory, the quantization process is considered as a typical uniform quantizer characteristic, which means, the unquantized signal is rounded to the nearest quantization level.

In general, two's complement (2'sc) binary is used to represent the physical quantized signal. The most significant bit (MSB) of the 2'sc binary is a sign bit and the remaining bits can be viewed as fractional bits. In other words, a binary fractional point is located in the middle of the two MSB bits. Therefore, the value of a B-bit 2'sc fraction is equivalent to $-a_0 2^0 + a_1 2^{-1} + a_2 2^{-2} + \cdots + a_B 2^{-(B-1)}$. The relationship between the 2'sc representation and the quantized level depends on the full-scale level V_m of the quantizer, for instance, 5-volt. Therefore, the step-size Δ and the nth quantization error $e_{q,n}$ of the quantizer are expressed as $\Delta = 2V_m/2^B$ and $e_{q,n} = \hat{x}_n - x_n$, respectively.

According to the statistical representations of quantized error [3], the amplitude of quantization noise of a quantizer is equivalent to $-\frac{\Delta}{2} < e_{q,n} \leq \frac{\Delta}{2}$. The mean value of e_q is zero since the e_q is assumed to be a uniform distribution and a white-noise sequence. Therefore, the noise variance of B-bit quantizer can be derived as

$$\sigma_{e_q}^2 = \frac{\Delta^2}{12} = \frac{(2V_m/2^B)^2}{12} \tag{1}$$

In addition, the signal-to-quantization noise ratio (SQNR) of B-bit quantizer can be expressed as

$$SQNR = 10 \cdot log_{10}\left(\sigma_s^2/\sigma_{e_q}^2\right) \tag{2}$$

where σ_s^2 is the average signal power. In view of a sine-wave with amplitude V_m, $\sigma_s^2 = (V_m/\sqrt{2})^2$. Therefore, Eq. (2) can be further simplified as

$$SQNR = 6.02B + 1.76 \text{ dB} \tag{3}$$

It is clear that the SQNR is proportional to the number of bits.

2.2. **Format of Finite Wordlength.** Without loss of generality, a format of finite wordlength is defined as $< S_b, I_b, F_b >$, where S_b, I_b and F_b express the sign-, the integer- and the fraction-bit, respectively. The total wordlength B is equivalent to the summation of S_b, I_b and F_b. For digital signal processing, both the sign- and the integer-bit can reveal the signal power and the signal amplitude. In addition, the fraction-bit is employed to preserve the signal precision.

In order to easily analyze the finite wordlength, the format of finite wordlength should be revised by the left-shift of fractional point with I_b-bit. Therefore, Eq. (1) can be applied with degenerating $10 \cdot log(2^{I_b})^2$ dB for the left-shift operation and, further, the quanitzed noise power of left-shift wordlength can be described as

$$\sigma_{e_{q,LSH}}^2 = \frac{\sigma_{e_q}^2}{(2^{I_b})^2} \tag{4}$$

Obviously, the total bit number of left-shift version is the same as that of un-left-shift version.

2.3. **Restrictive SNR Degradation.** In order to obtain a cost-effective hardware design, the **finite wordlength analysis** of the received signal path for OFDM receiver has to be derived to meet the system performance requirement. Therefore, a **restrictive SNR degradation** is defined as the physical system SNR degradation (loss) between the infinite- and finite-wordlength effect on OFDM transceiver. In practice, the simulation results of floating- (FLT) and fixed-point (FXP) can be expressed as the SNR loss of infinite- and finite-wordlength for OFDM transceiver, respectively, and given as

$$SNR_{q,\,deg} = SNR_{FLT} - SNR_{FXP} \tag{5}$$

where SNR_{FLT} and SNR_{FXP} denote the averaged output SNR for all subchannels in FLT- and FXP-simulation, respectively. Furthermore, both SNR_{FLT} and SNR_{FXP} can be obtained as

$$SNR = \frac{\sum_{k=-K/2,k\neq 0}^{K/2} \mathcal{S}_k}{\sum_{k=-K/2,k\neq 0}^{K/2} \mathcal{E}_k} \tag{6}$$

where k is the subcarrier index and K is the total used subcarriers. The $\mathcal{S}_k$ and $\mathcal{E}_k$ are the signal power and the error power, respectively, on the kth subcarrier. For an additive white Gaussian noise (AWGN) channel, the design criterion of the restrictive SNR degradation induced by quantization noise has to satisfy the following condition, specified as

$$SNR_{q,\,deg} \leq 0.25 \ \text{dB} \tag{7}$$

Considering the related system performance, requirement and parameter including the uncoded BER of 10^{-6} and the required SNR of 26 dB for 64-QAM modulation, the design criterion, as described in Eq. (7), for OFDM transceiver is true iff the system SQNR should be larger than and equivalent to 38 dB. In other words, the standard deviation of quantization noise σ_q should be less than and equal to $\frac{1}{4}$ of standard deviation of AWGN σ_v, i.e., $\sigma_q \leq \frac{1}{4}\sigma_v$. Besides, under the assumption as described in Eq. (7), the symbol error rate (SER) is around 10^{-4}.

3. **Finite Wordlength Analysis of ADC.** For a baseband receiver as shown in Fig. 1, the required bit number of ADC is determined by the following crucial factors [4][5][6][7]:

1. Required SNR (or BER) based on modulation type
2. Restrictive SNR degradation caused by quantization noise
3. Peak-to-average ratio (PAR) resulted from constellation size
4. Over-sampling ratio (OSR) of ADC

According to these crucial factors, Eq. (3) can be revised and hence the bit number of ADC can be formulated as

$$B_{ADC} = \frac{1}{6.02} \cdot [SQNR + PAR_{OFDM} - 1.76 - 10 \cdot log(2 \cdot OSR)] \tag{8}$$

where $SQNR$ can be acquired from the required SNR (or BER) and the restrictive SNR degradation as described in pervious subsection. In addition, OSR [5] is defined as $OSR = \frac{f_s/2}{f_B}$, where f_s and f_B express the sampling rate of ADC and the signal BW, respectively. Significantly, the SQNR can be improved by 3 dB for each doubling of OSR. PAR_{OFDM} is described as the **peak-to-average voltage ratio** of OFDM system, i.e., **peak-to-root mean square (RMS) voltage ratio**.

3.1. **PAR.** In light of OFDM system, the peak voltage of transmitting signal is appearance at the peak sum of all subchannels since the modulation scheme for all subchannels is identical, such as BPSK, QPSK, 16-QAM or 64-QAM. Similarly, the average power for all subchannels is equivalent to the summation of average power for all subchannels. Therefore, the total average power for all subchannels can be formulated as

$$\mathbf{P}_{OFDM} = \sum_{k=-K/2,k\neq 0}^{k=K/2} \mathbf{P}_k = K \cdot \mathbf{P}_{sub} \tag{9}$$

where $\mathbf{P}_k$ is the average power of the kth subchannel. $\mathbf{P}_{sub}$ is the subchannel average power and then $\mathbf{P}_{sub} = \mathbf{P}_k \ \forall \ k \in [-K/2, K/2]$ and $k \neq 0$. Furthermore, based on the definition of peak-to-RMS voltage ratio, the PAR_{OFDM} [4] can be derived as

$$\begin{aligned} PAR_{OFDM} &= \frac{\mathbf{V}_p}{\sqrt{\mathbf{P}_{OFDM}}} = \frac{\sum_{k=-K/2,k\neq 0}^{k=K/2} \mathbf{V}_k}{\sqrt{K \cdot \mathbf{P}_{sub}}} \\ &= \frac{1}{\sqrt{K}} \sum_{k=-K/2,k\neq 0}^{k=K/2} \frac{\mathbf{V}_k}{\sqrt{\mathbf{P}_{sub}}} = \frac{1}{\sqrt{K}} \sum_{k=-K/2,k\neq 0}^{k=K/2} PAR_k \end{aligned} \quad (10)$$

where $\mathbf{V}_p$ is the peak voltage of OFDM system and $\mathbf{V}_k$ denotes the peak voltage of the kth subchannel. Significantly, the quantity of $\mathbf{V}_k$ is dependent on the modulation scheme since the peak value is different for various constellation. The PAR_k is defined as the peak-to-RMS voltage ratio on the kth subchannel. Considering a particular condition that the modulation scheme for all subchannel is identical, the PAR of OFDM system can be further simplified as

$$PAR_{OFDM} = \sqrt{K} \cdot PAR_{sub} = \sqrt{K} \cdot \frac{\mathbf{V}_{peak}}{\sqrt{\mathbf{P}_{sub}}} \quad (11)$$

where PAR_{sub} stands for the subchannel PAR, i.e., $PAR_{sub} = PAR_k \ \forall \ k \in [-K/2, K/2]$ and $k \neq 0$. Similarly, $\mathbf{V}_{peak}$ represents the subchannel peak amplitdue and, thus, $\mathbf{V}_{peak} = \mathbf{V}_k \ \forall \ k \in [-K/2, K/2]$ and $k \neq 0$. The maximum $\mathbf{V}_{peak}$ occurs at the 64-QAM of all constellations as illustrated in Fig. 2.

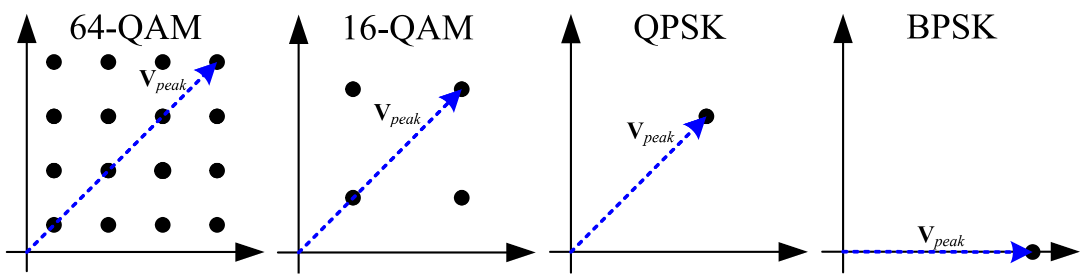


FIGURE 2. Peak values for various constellation schemes.

3.2. **Bit Number of ADC.** Considering the system specifications as shown in Table 1, the sampling rate f_s of ADC is 40 MHz and the signal BW f_B is 20 MHz. Therefore, the OSR is equivalent to 1, namely, 3 dB. The system SQNR should be $\geq$ 38 dB based on the restrictive SNR described in Eq. (7). The PAR_{OFDM} can be derived as

$$PAR_{OFDM} = \frac{1}{\sqrt{K}} \cdot (N_d \cdot PAR_{64-QAM} + N_p \cdot PAR_{BPSK}) \quad (12)$$

where $N_d = 48$, $N_p = 4$ and $K = 52$. PAR_{64-QAM} and PAR_{BPSK} denote the PARs of 64-QAM and BPSK, respectively. Based on the peak value and the average power, $PAR_{64-QAM} = 1.53$, $PAR_{BPSK} = 1$ and, further, $PAR_{OFDM} = 20.62$ dB. Therefore, the bit number of ADC can be acquired as

$$B_{ADC} = \frac{1}{6.02} \cdot [38 + 20.62 - 1.76 - 3] \approx 8.95 \quad (13)$$

In conclusion, the bit number of ADC is equivalent to 9-bit. The format of finite wordlength is $< 1, 0, 8 >$ and, thus, $-1.0 \leq$ the amplitdue of ADC output $\leq (1 - 2^{-8})$.

4. **Receiving Filter.** Both the quantized coefficient and the PAR of receiving filter are important issues to determine the finite wordlength of receiving filter. Actually, the effect of quantized coefficient error gives raise to a deviation of frequency response from the desired response. The output dynamic range of receiviving filter is dominated by the PAR of receiving filter. The input finite wordlength of receiving filter is directly from the output of ADC.

In this OFDM transceiver as shown in Fig. 1, the transmitting and the receiving filters are designed as an interpolation and a decimation filter with the up- and down-sampling rate of 2, respectively. The finite-wordlength version of receiving filter has to satisfy the specifications listed as follows:

- Passband ripple: $\delta_p \leq 0.25$ dB
- Stopband attenuation: $\delta_s \geq 32$ dB
- Passband edge frequency: 8.4375 MHz
- Stopband edge frequency: 10.5 MHz

According to the specification, the finite-impulse response (FIR) filter is adopted to realize the receiving filter with the tap-number N_{Tap} of 35. The design considerations for both the quantized coefficient and the PAR of the receiving filter are described in the following subsections.

4.1. **Quantized Coefficient.** In order to maintain the spectrum consistence between FLT- and FXP-version, the quantized error should be considered in the design of quantized coefficient for the receiving filter and thus modeled as

$$h_{q,m} \quad = \quad h_m + \Delta h_{q,m}, \;\; m = 0, 1, \;\ldots, (N_{Tap} - 1) \tag{14}$$

where h_m is the mth coefficient of receiving filter. In addition, $h_{q,m}$ and $\Delta h_{q,m}$ express the quantized-coefficient and the quantized-coefficient error of the mth tap, respectively. The corresponding frequency representation of receiving filter can be represented as

$$H_q(w) \quad = \quad H(w) + \Delta H_q(w) \tag{15}$$

where $H_q(w)$ and $H(w)$ are the frequency responses of quantized- and unqunatized-coefficient. $\Delta H_q(w)$ is the frequency response of quantized-coefficient error and $\Delta H_q(w) = \sum_{m=0}^{N_{Tap}-1} \Delta h_{q,m} \cdot e^{-jwm}$. Therefore, the design of quantized coefficient has to follow the useful design guide that the amplitude of $\Delta H_q(w)$ should be limited to meet the specifications of receiving filter. In view of a direct form FIR filter with rounding coefficients, all of quantized-coefficient errors are assumed to be uniform distribution and zero mean. Then, the statistical error bound [8] is given by

$$\Delta H_q(w) \quad = \quad 2^{-B_{h_q}} \cdot \sqrt{\frac{N_{Tap}}{3}} \tag{16}$$

where B_{h_q} is the bit number of quantized-coefficient for receiving filter.

Considering the specification of receiving filter, the stopband attenuation is regarded as the critical factor of limiting deviation of receiving filter. In other words, Eq. (16) should be less than and equal to the stopband attenuation. Therefore, the coefficient bit number of receiving filter can be obtained as

$$20 \cdot log \left(2^{-B_{h_q}} \cdot \sqrt{\frac{N_{Tap}}{3}} \right) \leq -\,32\,\text{dB} \quad \text{and} \quad B_{h_q} \geq 7.09 \tag{17}$$

According to the result of Eq. (17), the bit number of quantized-coefficient for received filter is 8, namely, $< 1, 0, 7 >$. However, both the passband ripple and the stopband attenuation with the format $< 1, 0, 7 >$ coefficients can not meet the specification as shown in Fig. 3 (a). Hence, the fraction-bit of quantized-coefficient should be added an additional bit to be $< 1, 0, 8 >$. Then, both the passband ripple and the stopband attenuation can meet the specification as illustrated in Fig. 3 (b). The impulse response of receiving filter and its corresponding coefficients are illustrated in the left- and right-side of Fig. 4.

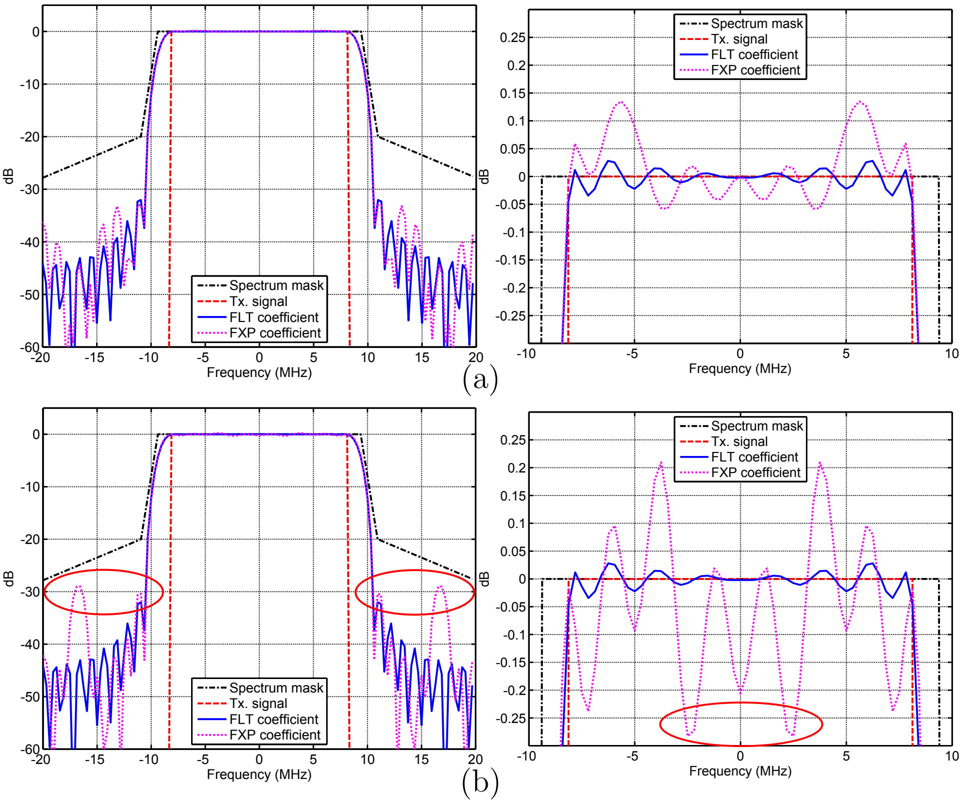
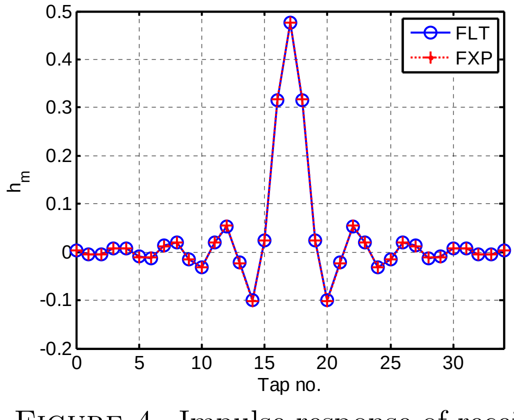


FIGURE 3. Spectrum mask, spectrum allocation of transmitted signal, frequency response of received filter with FLT- and FXP-coefficient. (a) $< 1, 0, 7 >$ and (b) $< 1, 0, 8 >$.



h_m	Coefficient FLT	Coefficient FXP	h_m	Coefficient FLT	Coefficient FXP
$h_0 = h_{34}$	0.0035752	0.00390625	$h_9 = h_{25}$	-0.016214	-0.01562500
$h_1 = h_{33}$	-0.0043545	-0.00390625	$h_{10} = h_{24}$	-0.032515	-0.03125000
$h_2 = h_{32}$	-0.0051676	-0.00390625	$h_{11} = h_{23}$	0.019164	0.01953125
$h_3 = h_{31}$	0.0068340	0.00781250	$h_{12} = h_{22}$	0.053653	0.05468750
$h_4 = h_{30}$	0.0080236	0.00781250	$h_{13} = h_{21}$	-0.021535	-0.02343750
$h_5 = h_{29}$	-0.0097720	-0.01171875	$h_{14} = h_{20}$	-0.099774	-0.10156250
$h_6 = h_{28}$	-0.0127670	-0.01171875	$h_{15} = h_{19}$	0.023073	0.02343750
$h_7 = h_{27}$	0.0129840	0.01171875	$h_{16} = h_{18}$	0.316140	0.31640625
$h_8 = h_{26}$	0.0203430	0.01953125	h_{17}	0.476390	0.47656250

FIGURE 4. Impulse response of receiving filter and its corresponding coefficient.

4.2. **PAR.** Another critical factor of receiving filter is PAR (PAR_{RxFLR}) [4] since the output dynamic range of receiving filter could be enlarged by this factor. Actually, the PAR_{RxFLR} is used to describe the peak-to-RMS ratio at the output of receiving filter. The parameter PAR_{RxFLR} can be employed to determine the output integer-bit of receiving filter and, thus, defined as

$$PAR_{RxFLR} = \frac{\mathbf{V}_{O,RxFLR}}{\mathbf{RMS}_{O,RxFLR}} \tag{18}$$

where $\mathbf{V}_{O,RxFLR}$ and $\mathbf{RMS}_{O,RxFLR}$ express the output peak amplitude and the output RMS amplitude of receiving filter, respectively. The output peak amplitude of receiving filter can be derived as

$$\mathbf{V}_{O,RxFLR} = \mathbf{V}_{I,RxFLR} \cdot \sum_{m=0}^{N_{Tap}-1} |h_m| \quad (19)$$

where $\mathbf{V}_{I,RxFLR}$ is the maximum input peak amplitude of receiving filter. It is obvious that the $\mathbf{V}_{I,RxFLR}$ results from the peak value of 64-QAM on each subchannel as illustrated in Fig. 2. Furthermore, the output RMS amplitude of receiving filter can be acquired as

$$\mathbf{RMS}_{O,RxFLR} = \sqrt{\mathbf{E}\left[\left(\sum_{m=0}^{N_{Tap}-1} h_m \cdot y_{n-m}\right)^2\right]} = \sqrt{\mathbf{E}\left[y_n^2\right] \cdot \sum_{m=0}^{N_{Tap}-1} h_m^2} \quad (20)$$

where $\mathbf{E}[\cdot]$ is expectation operator. y_n is the nth input sample of receiving filter, namely ADC output. Therefore, based on Eq. (19) and (20), the PAR_{RxFLR} is equivalent to 2.53. It is clear that the input of receiving filter will be enlarged 2.53 times at the output of receiving filter. Therefore, the sign- and the integer-bit of receiving filter output are 1- and 2-bit, respectively.

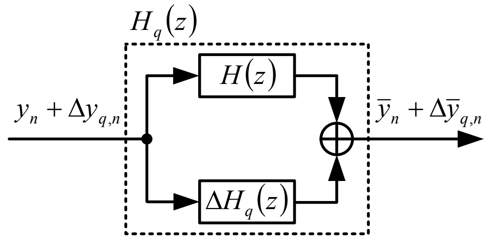


FIGURE 5. Representation of the quantized receiving filter.

The final issue is the output fraction-bit number of receiving filter. Based on Eq. (15), the representation of quantized receiving filter is illustrated in Fig. 5, where z is the z-transform operator. $H(z)$ is the transfer function of FLT-version of receiving filter. Besides, $H_q(z)$ and $\Delta H_q(z)$ are the transfer functions of quantized and quantized-error for receiving filter, respectively. Both the input and the output of receiving filter with individual quantized noise are denoted as $y_n + \Delta y_{q,n}$ and $\bar{y}_n + \Delta\bar{y}_{q,n}$, respectively. Furthermore, the signal powers (or variances) of both input and output of the quantized received filter can be expressed as $\sigma^2_{y_n} + \sigma^2_{y_{q,n}}$ and $\sigma^2_{\bar{y}_n} + \sigma^2_{\Delta\bar{y}_{q,n}}$, respectively, since the quantized error sequence is uncorrelated with the unquantized sequence. Based on the Parseval's theorem [3], the power of the quantized noise can be derived as

$$\begin{aligned}\sigma^2_{\Delta\bar{y}_{q,n}} &= \left(\sigma^2_{y_n} + \sigma^2_{\Delta y_{q,n}}\right) \cdot \left[\frac{1}{2\pi j} \oint_{|z|=1} \Delta H_q(z)\, \Delta H_q\left(z^{-1}\right) \frac{dz}{z}\right] \\ &\quad + \sigma^2_{\Delta y_{q,n}} \cdot \left[\frac{1}{2\pi j} \oint_{|z|=1} H(z)\, H\left(z^{-1}\right) \frac{dz}{z}\right] \\ &= \sigma^2_{y_n} \sum_{m=0}^{N_{Tap}-1} |\Delta h_{q,m}|^2 + \sigma^2_{\Delta y_{q,n}} \sum_{m=0}^{N_{Tap}-1} |h_m|^2 + \sigma^2_{\Delta y_{q,n}} \sum_{m=0}^{N_{Tap}-1} |\Delta h_{q,m}|^2 \quad (21)\end{aligned}$$

where the third term of Eq. (21) can be further ignored since this term is too small compared with the others. Significantly, the terms in the square brackets can be regarded as the system power gains caused by $H(z)$ and $\Delta H_q(z)$. Therefore, the main sources of quantized noise at the receiving filter output come from the input signal and the input quantized noise passing through $\Delta H_q(z)$ and $H(z)$, respectively. Therefore, the first and the second terms are equivalent to

$$\sigma_{y_n}^2 \cdot \sum_{m=0}^{N_{Tap}-1} |\Delta h_{q,m}|^2 = 5.73\times 10^{-5} \quad \text{and} \quad \sigma_{\Delta y_{q,n}}^2 \cdot \sum_{m=0}^{N_{Tap}-1} |h_m|^2 = 1.17\times 10^{-6} \tag{22}$$

where $\sum_{m=0}^{N_{Tap}-1} |\Delta h_{q,m}|^2 = 8.1\times 10^{-5}$ and $\sum_{m=0}^{N_{Tap}-1} |h_m|^2 = 0.92$. Besides, $\sigma_{y_n}^2 = 0.707$ and $\sigma_{\Delta y_{q,n}}^2 = 1.27\times 10^{-6}$ since $B_{ADC} =$ 9-bit and $-1.0 \leq$ amplitdue of ADC output $\leq (1-2^{-8})$. Finally, $\sigma_{\Delta \bar{y}_{q,n}}^2 = 5.84\times 10^{-5}$. According to the results of Eq. (4) and (22), the bit number of receiving filter output can be acquired as

$$\frac{\sigma_{\Delta \bar{y}_{q,n}}^2}{(2^{I_{b,RxFLR}})^2} = \frac{1}{12} \cdot \left(\frac{2V_{m,RxFLR}}{2^{B_{O,RxFLR}}} \right)^2 \tag{23}$$

where $I_{b,RxFLR} = 2$, $V_{m,RxFLR} = 2.53$ and therefore $B_{RxFLR} \approx 9.57$. The total bit number of receiving filter output $B_{O,RxFLR}$ is 10 at least, namely $< 1, 2, 7 >$. However, the coefficients of h_0 and h_{34} are 2^{-8}, as well as the coefficients of h_1, h_2, h_{32} and h_{33} are -2^{-8}. All of these taps are useless while the absolute values of the real- and imaginary-part of y_{n-m} are less than 1. Actually, the absolute values of $\Re(y_{n-m})$ and $\Im(y_{n-m})$ are frequently less than 1 caused by the attenuation of multipath frequency-selective fading channel. Practically, considering the effect of multipath frequency-selective fading channel, the fraction-bit of receiving filter output is added 2-bit to be 9-bit to reserve the signal precision. Hence, $B_{O,RxFLR} = 12$, i.e., $< 1, 2, 9 >$.

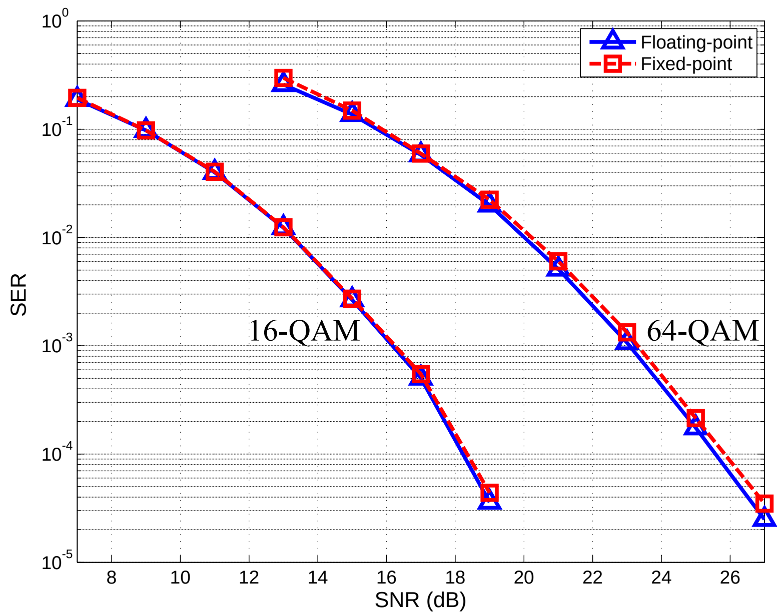


Figure 6. SERs of 16- and 64-QAM modulation schemes for OFDM transceiver as illustrated in Fig. 1.

5. **Numerical Simulation Results.** A C model is developed to emulate the OFDM transceiver as shown in Fig. 1 for demonstrating the finite wordlength and the system performance. The statistical model [10] is referred to establish the multipath frequency-selective fading channel with RMS delay spread of 100 ns for a large open space [9]. In the C model, the fixed-point wordlength of ADC output is 9-bit, which is the same as that

of the input of receiving filter. The fixed-point wordlengthes of coefficient and output for receiving filter are 9- and 12-bit, respectively.

The transmitting and the receiving filters are a interpolation and a decimation filters with the up- and down-sampling factor of 2, respectively. Therefore, the input and the output sampling rate of transmitting filter are 20 and 40 MHz, respectively. On the contrary, the input and the output sampling rate of receiving filter are 40 and 20 MHz, respectively.

The number of simulated subchannel symbols is more than 5×10^4, which is corresponding to 1045 OFDM symbols. Significantly, the design target of restricative SNR degradation ($\leq$0.25 dB) is derived under AWGN channel in finite wordlength analysis. The SERs of 16- and 64-QAM modulation schemes over the multipath frequency selective fading channel are illustrated in Fig. 6. For given SNR=26 dB and SER$\approx10^{-4}$, the system SNR degradation is less than 0.3 dB, which is close to the design target of restrictive SNR degradation.

6. **Conclusions.** The finite-wordlength analysis of ADC and receiving filter for OFDM baseband transceiver is presented in this paper. In order to obtain a cost-effective hardware design of baseband transceiver, the restrictive SNR degradation is defined as the design criterion of finite-wordlength analysis with the corresponding system performance indexes, quantized noises and system parameters including BER, SNR, SQNR, PAR, etc. According to the finite-wordlength analysis, the bit number of ADC is 9-bit. The bit numbers of coefficient and output of receiving filer are 9- and 12-bit ,respectively. For the multipath frequency selective fading channel, the simulation results of OFDM transceiver show that the system SNR degradation between FLT- and FXP-version is less than 0.3 dB, which is close to the design criterion ($\leq$0.25 dB) under AWGN channel in finite-wordlength analysis.

REFERENCES

[1] *IEEE Std 802.11a-1999(R2003)*, Jun. 2003.

[2] C. F. Wu, M. T. Shiue and C. K. Wang, Joint Carrier Synchronization and Equalization for Packet-Based OFDM Systems over Multipath Fading Channel, *IEEE Trans. on Vehicular Technology,* vol. 59, no. 1, pp-248-260, Jan. 2010.

[3] A. V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Processing*, Englewood Cliffs, Prentice-Hall, 1998.

[4] W. Y. Chen, *DSL: Simulation Techniques and Standards Development for Digital Subscriber Line Systems*, Macmillan Tech. Pub., 1998.

[5] J. Khoury and H. Tao, Data Converters for Communication Systems, *IEEE Comm. Magazine*, vol. 36, no. 10, pp. 113–117, Oct. 1998.

[6] J. A. C. Bingham, *ADSL, VDSL, and Multicarrier Modulation*, John Wiley & Sons, Inc., 2000.

[7] T. Starr, J. M. Cioffi and P. J. Silverman, *Understanding Digital Subscriber Line Technology*, Prentice Hall, Inc., 1999.

[8] E. C. Ifeachor and B. W. Jervis, *Digial Signal Processing – A Practical Approach*, Addison-Wesley Pub., 1st ed., 1993.

[9] R. van Nee, and R. Prasad, *OFDM for Wireless Multimedia Communication*, Artech House, Inc., 2000.

[10] A. A. M. Saleh, R. A. Valenzuela, "A Statistical Model for Indoor Multipath Propagation," *IEEE J. on Selected Areas in Comm.*, vol. SAC-5, no. 2. pp. 128–137, Feb. 1987.