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Lossy Compression for Compressive Sensing of Three-Dimensional Images

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Abstract. *Tensor Compressive Sensing (CS) is an emerging approach for higher-order data representation applications, such as medical imaging, video sequences, and multi-sensor networks. Because the characteristics of CS acquisition differ considerably from those of traditional image acquisition, traditional image compression solutions may not be particularly applicable. In this paper, we propose a tensor-based coding scheme for Three-dimensional (3D) images. To establish this scheme, we first designed a 3D CS method via Tucker decomposition to secure favorable reconstruction quality for the acquired 3D images. Next, we designed a tensor quantization for the obtained 3D CS measurements that works without transforming the tensor into vectors. Finally, we applied the methods in the Compressive Sensing Imaging (CSI) system for efficient encoding of the 3D images. We confirmed through a series of experiments that the proposed method outperforms other similar methods in terms of both reconstruction accuracy and processing speed.*
Keywords: Compressive sensing, Lossy compression, 3D image

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1. **Introduction.** Compressed Sensing (CS), the theory of which is based on the observation that various cases of natural signals are approximately sparse with respect to certain bases or frames has garnered considerable attention from researchers and developers in recent years [1-4]. To allow for digital transmission and storage of CS measurements, the measurements must be accurately quantified-that is, the measurements must be represented by finite symbols from a finite alphabet. The most intuitive method of quantifying measurements maps each of them to the closest element from the alphabet [5-6].

In practical CSI applications, CS acquisition is assumed to be implemented in some type of analog image-acquisition hardware such as a single-pixel camera. The acquired CS measurements are real-valued, which creates a large amount of data that must be stored and transmitted. The lossy compression of CS measurements is thus a necessary part of the CS acquisition process [7]. Most approaches to lossy CS measurement compression focus on problems involving 1D signals or 2D image data encoded in vectors, however, many important applications involve higher-order signals (e.g., 3D videos) [8-10].

Designing an efficient lossy compression of 3D CS acquisitions requires that two main questions to be addressed: How does one represent the image signal for the best possible CS reconstruction? Also, how does one efficiently quantify the 3D CS measurements? The main contribution of this paper is an efficient lossy compression solution for 3D CS acquisition of images in the CSI system. We developed the proposed solution by considering both the low-rank representation of images and quantization of CS measurements (i.e., the two questions mentioned above). In brief, in this study, we 1) designed a 3D CS algorithm by introducing a new Tucker decomposition algorithm and 2) designed a tensor quantization for the 3D CS measurements.

The remainder of this paper is organized as follows. Section 2 explains the theoretical background and describes the methods we used to validate the proposed approach. Section 3 presents a comparison among the reconstruction results of the proposed method and other existing methods, and Section 4 contains a brief summary of our contribution and conclusion.

To facilitate the distinction between scalars, vectors, matrices, and higher-order tensors, the type of a given quantity is reduced here by its representation: scalars are denoted by lower-case letters (x), vectors are written as capitals ($\mathbf{x}$), matrices corresponding to bold-face capitals ($\mathbf{X}$) and tensors are written as calligraphic letters ($\mathcal{X}$).

2. **TCS-based lossy compression.** In traditional signal acquisition systems, analog signals are often low-pass filtered to limit their bandwidth based on the Shannon-Nyquist sampling theorem prior to acquisition. The reconstruction quality can be improved by minimizing aliasing effect, which is caused by unlimited signal bandwidth of the signal [6]. In a CS acquisition system, reconstruction quality degrades due to the vectorization of multidimensional signals: futher, this is rather time consuming and not applicable in practice. Below, we introduce a tensor-based method of avoiding vectorization for CS acquisition in the CSI system that can be used to improve reconstruction quality. We designed a tensor quantization method which extends traditional vector quantization into its 3D version for coding, as also discussed below. The entire system, as shown in Fig.1, can be divided roughly into two parts: encoding and decoding.

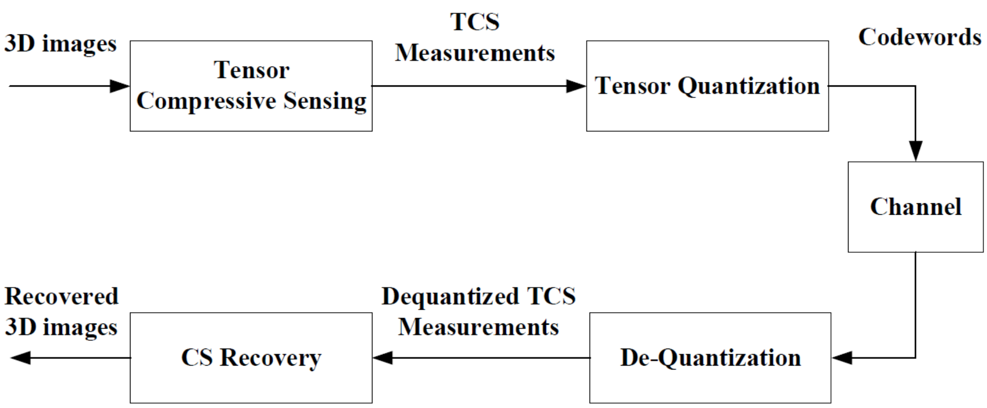

FIGURE 1. 3D CSI system framework designed via proposed method

2.1. **Encoding.** Let $\mathcal{X} \in R^{N_1 \times N_2 \times N_3}$ and assume that $\Phi_i \in R^{N_i \times M_i}$ $(N_i > M_i)$satisfies the NSP properties. Define the following

$$\mathcal{S} = \mathcal{X} \times_1 \Phi_1^T \times_2 \Phi_2^T \times_3 \Phi_3^T \tag{1}$$

where $\mathcal{S} \in \{s_{i_1,i_2,i_3}\}$, $i_1 = 1, \cdots, M_1; i_2 = 1, \cdots, M_2; i_3 = 1, \cdots, M_3$ and "$\times_k$" is the d-mode multiplication of HOSVD[11-12].

Let $\mathcal{C} \in \{c_{j_1,j_2,j_3}\}$, $i_1 = 1, \cdots, m_1; i_2 = 1, \cdots; m_2; i_3 = 1, \cdots, m_3$ represent the codebook and P_{j_1,j_2,j_3} be the encoding region associated with codeword c_{i_1,i_2,i_3}. Let $\mathcal{P} \in \{P_{j_1,j_2,j_3}\}$ denote the partition of the space. If the source s_{i_1,i_2,i_3} is in the encoding region P_{j_1,j_2,j_3} , its approximation is c_{j_1,j_2,j_3} and:

$$Q(s_{i_1,i_2,i_3}) = c_{j_1,j_2,j_3}, if s_{i_1,i_2,i_3} \in P_{j_1,j_2,j_3}, \tag{2}$$

Assuming a squared-error distortion measure, the average distortion is given by:

$$D_{i_1,i_2,i_3,j_1,j_2,j_3} = \|s_{i_1,i_2,i_3} - c_{j_1,j_2,j_3}\|^2 \tag{3}$$

Let (j_1^*, j_2^*, j_3^*) be the index which achieves the minimum, and set:

$$(j_1^*, j_2^*, j_3^*) = argminD_{i_1,i_2,i_3,j_1,j_2,j_3} \tag{4}$$

Update the code word as follows:

$$C_{j_1,j_2,j_3}^{(l+1)} = \frac{\sum_{Q(s_{i_1,i_2,i_3})=c_{j_1,j_2,j_3}^{(l)}} s_{i_1,i_2,i_3}}{\sum_{Q(s_{i_1,i_2,i_3})=c_{j_1,j_2,j_3}^{(l)}} 1} \tag{5}$$

This condition implies that the code tensor is the average of all those training tensors located in the encoding region. During implementation, one must ensure that at least one training tensor belongs to each encoding region.

The pseudo-code used for tensor-based encoding is presented in Algorithm 1.

```
Algorithm 1
Input:
(1) 3D image 𝒳
(2) Sensing matrices Φ1, Φ2, Φ3;
(3) ε
Output:
Code tensor 𝒞
Start:
(1) Compute 𝒮 according to formula (1) ;
(2) Generate 𝒞 ∈ R^{m1×m2×m3} randomly.
(3) for l = 1 : L, do:
        for i1 = 1 : M1, i2 = 1 : M2, i3 = 1 : M3, do:
          for j1 = 1 : m1, j2 = 1 : m2, j3 = 1 : m3, do:
            Compute D_{i1,i2,i3,j1,j2,j3} according to (3)
          end for
            Find the optimal index (j1*, j2*, j3*) and set Q(s_{i1,i2,i3}) = c^{(l)}_{j1,j2,j3}
        end for
        Update the all code words c^{(l+1)}_{j1,j2,j3}
        If (D^{(l-1)} − D^{(l)})/D^{(l-1)} > ε , break;
    end for
```

2.2. **Decoding.** De-quantify $\widetilde{\mathcal{S}}$, which is based on the quantization tensor $\mathcal{C}$. See the following:

$$\widetilde{s}_{i_1,i_2,i_3} = w_{i_1,i_2,i_3,j_1,j_2,j_3} \cdot c_{j_1,j_2,j_3} \tag{6}$$

where $w_{i_1,i_2,i_3,j_1,j_2,j_3} = \begin{cases} 1, if s_{i_1,i_2,i_3} \in P_{j_1,j_2,j_3} \\ 0, else \end{cases}$ are the weights.

Fig.2 Reconstruction of Foreman by KCS

FIGURE 2. Reconstruction of foreman by PC-CS

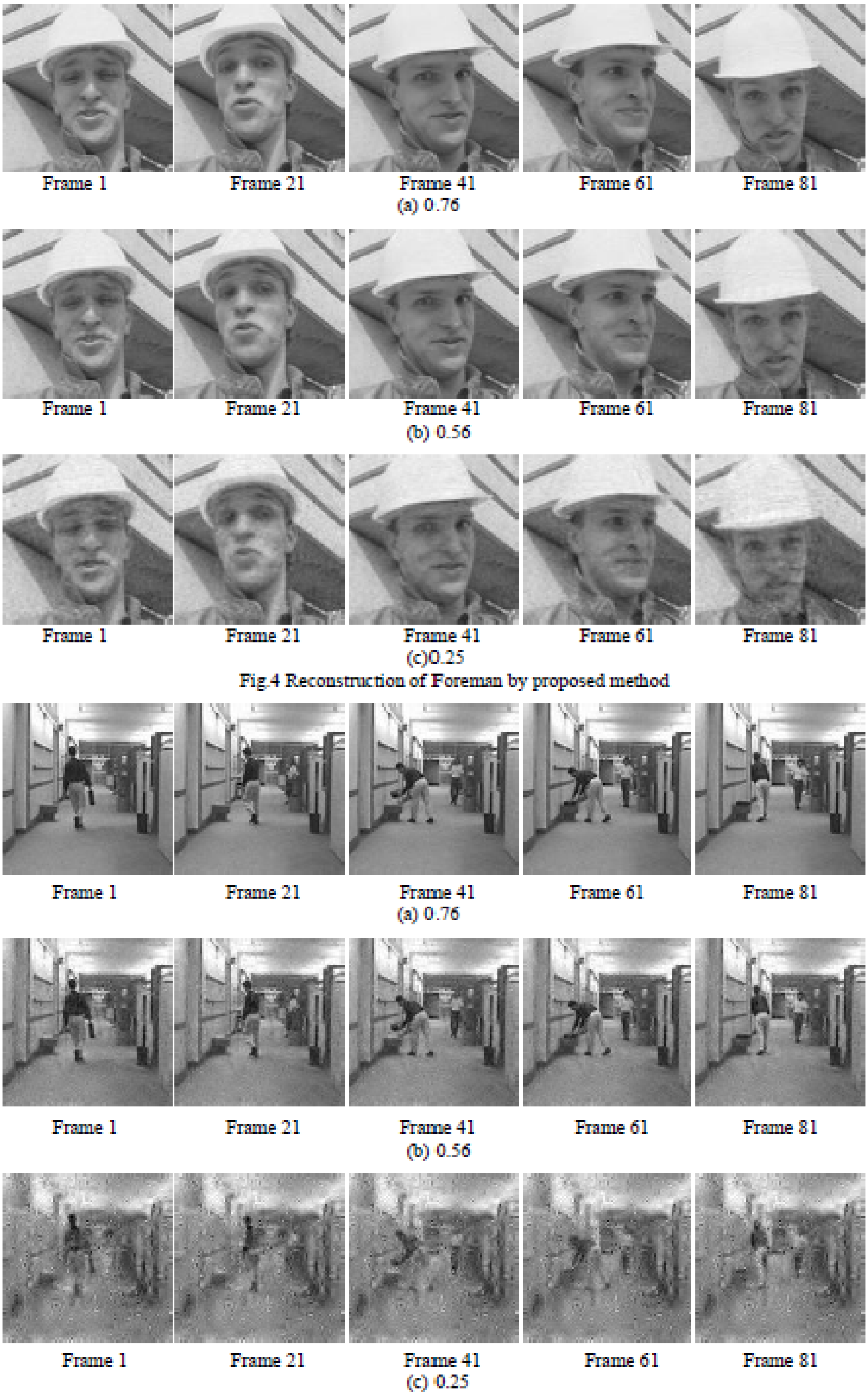

Figure 3. Reconstruction of mobile by KSC

To reconstruct $\widetilde{\mathcal{X}}$ from $\widetilde{\mathcal{S}}$ and Φ_d(d=1,2,3), we'll introduce the following tensor-based reconstruction algorithm [12]:

$$\widetilde{\mathcal{X}} = \mathcal{S} \times_1 (\mathbf{Z}_1\mathbf{S}^{\dagger}_{(1)}) \times_2 (\mathbf{Z}_2\mathbf{S}^{\dagger}_{(2)}) \times_3 (\mathbf{Z}_3\mathbf{S}^{\dagger}_{(3)}) \tag{7}$$

We assume that the following set of compressive multi-way measurements is available:

$$\mathbf{Z}^{(\mathrm{n})} = \mathcal{X} \times_1 \Phi_1 \times_2 \cdots \times_{\mathrm{n}-1} \Phi_{\mathrm{n}-1} \times_{\mathrm{n}+1} \Phi_{\mathrm{n}+1} \times_{\mathrm{n}+2} \cdots \times_{\mathrm{N}} \Phi_{\mathrm{N}} \tag{8}$$

and that

$$\mathbf{Z}_{(\mathrm{n})} = (\mathbf{Z}^{(\mathrm{n})})_{(\mathrm{n})} \tag{9}$$

where "† "stands for the pseudo-inverse of a matrix [12]. The pseudo-code used for tensor-based encoding is presented in Algorithm 2.

Algorithm 2
Input: (1) Code tensor $\mathcal{C}$ (2) Weight tensor $\mathcal{W}$ (3) Sensing matrices Φ_1, Φ_2, Φ_3 Output: $\widetilde{\mathcal{X}}$ Start: (1) De-quantizate $\widetilde{\mathcal{S}}$ according to formula (6) (2) Compute $\mathbf{Z}_{(\mathrm{n})}$ according to (8) and (9) (3) Reconstruct $\widetilde{\mathcal{X}}$ according to (7)

3. **Simulation Results and Analysis.** We experimentally demonstrated the performance of our TCS-based lossy compression methods by using it to reconstruct three 3D video sequences: "Foreman", "Mobile", and "Hall". We also reconstructed the same videos using two other state of the art methods, Kronecker-based Compressive Sensing (Kronecker CS) [13] and CANDECOMP/PARAFAC-based Compressed Sensing (CP-CS) [14], for the sake of comparison.

Each frame of the video sequence was preprocessed to 128128 pixels with 128 frames, the video data together was represented by a 128128128 tensor. The randomly constructed Gaussian measurement matrix for each mode was $128 \times M_d(d = 1, 2, 3)$ in size after preprocessing, and the code tensor was$m_1 \times m_2 \times m_3$, so the normalized number of samples was$\frac{m_1 \times m_2 \times m_3}{128^3}$.

3.1. **Reconstruction results.** Table 1 depicts the PSNR of the two 3D video sequences (to save space, only two of the three videos are shown) recovered by all the three methods with sampling rates of $0.76 \approx \frac{(117)^3}{(128)^3}$, $0.56 \approx \frac{(106)^3}{(128)^3}$, and $0.25 \approx \frac{(80)^3}{(128)^3}$. Figures 2-7 show the reconstruction performance of the three methods, where the proposed method well and clearly outperformed the other two.

3.2. **Computational complexity analysis.** We also compared the computational complexity of the three algorithms. The most complex procedure for reconstructing the original signal of KCS-based compression is Basis Pursuit (BP): the average number of BP iterations for all 3D samples was about 1000, and the most complex procedure of CP-CS-based compression was the twice 11-norm algorithm, which took an average of 79 iterations to reconstruct the original signal of all 3D samples. The proposed compression method, which is based on Tucker-TCS, does not involve iterations and as such is extremely fast. The computation time (specifically, the average computation time of the

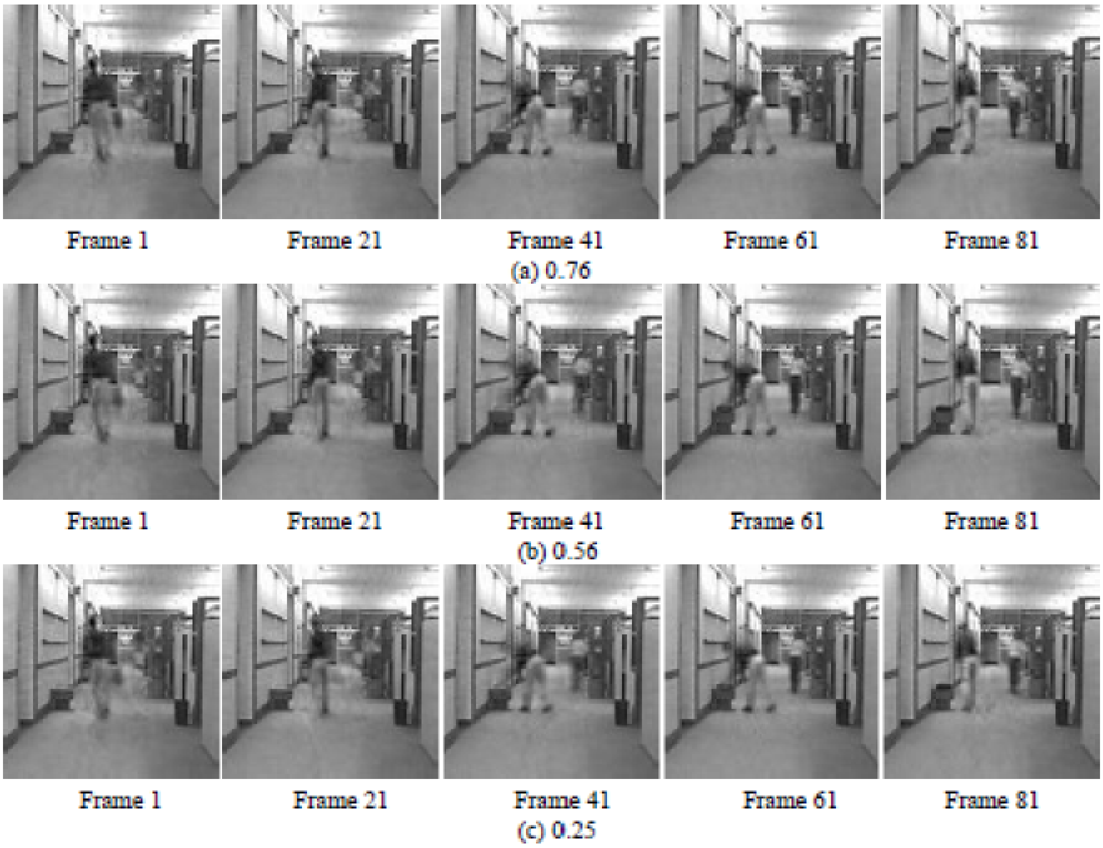

FIGURE 4. Reconstruction of mobile by PC-SC

experiments at all sampling ratios) required in each case confirmed that the proposed algorithm provides much faster computation than the other two, as shown in Table 2.

TABLE 1. PSNR (dB) comparison of the three methods

Sampling Rate	Methods	Foreman	Hall	Mobile
0.76	KCS	37.23	25.99	35.31
	CP-CS	24.96	19.57	30.80
	Proposed	43.34	31.42	45.53
0.56	KCS	30.69	21.21	27.14
	CP-CS	24.68	19.40	30.52
	Proposed	38.01	27.46	40.66
0.25	KCS	22.19	16.45	19.22
	CP-CS	24.02	18.92	29.50
	Proposed	31.22	22.68	33.88

TABLE 2. Comparison of computation time (s)

Methods	Foreman	Hall	Mobile
KCS	4334	4220	3419
CP-CS	2081	2070	1997
Proposed	2.89	2.62	2.77

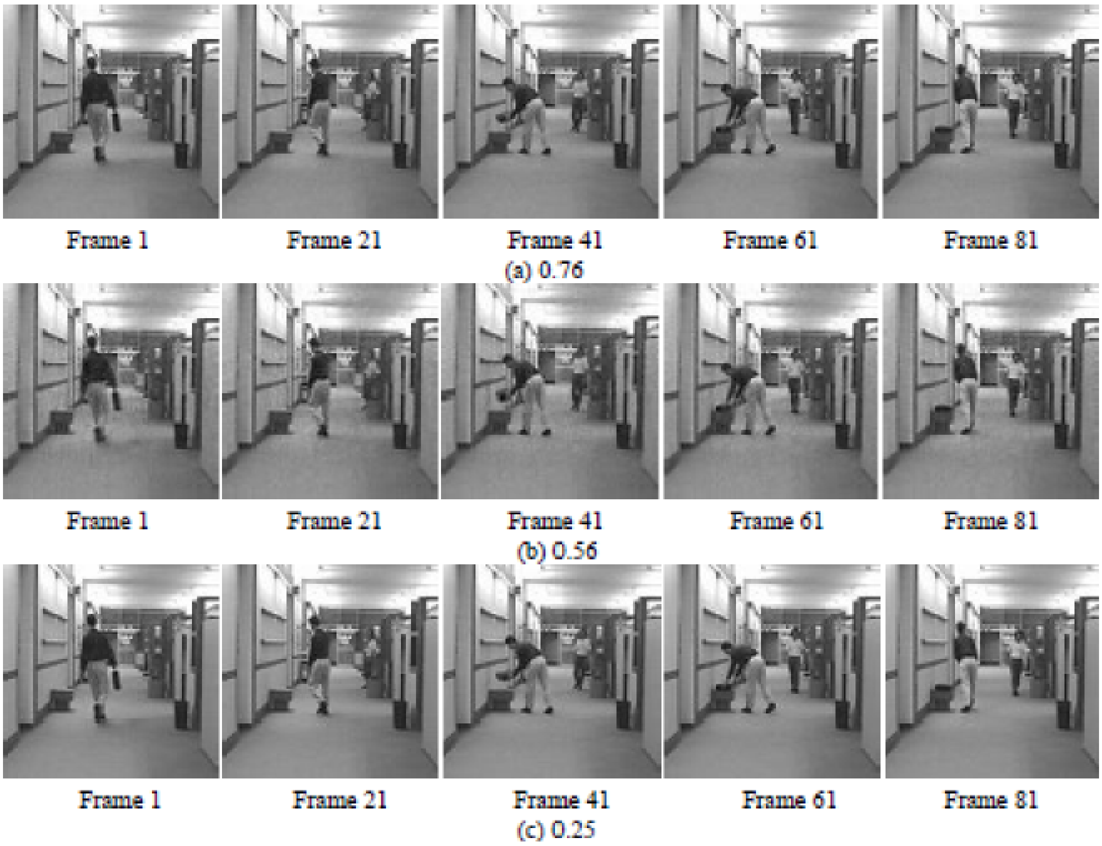

FIGURE 5. Reconstruction of mobile by the proposed method

4. **Conclusions.** In this study, we developed a tensor quantization method for efficient lossy compression of 3D CS acquisitions. Simulation results showed that the proposed solution achieves better performance and subjective quality of the CSI system compared to two other similar methods, with lower computational complexity.

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