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A Novel Super-Resolution Direction Finding Method for Wideband Signals Based on Multiple Small Aperture Subarrays

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Abstract. *The single small aperture subarray is hard to meet the requirement of the accuracy for multiple targets in super-resolution direction finding, to this problem, a new method for wideband signals based on partly calibrated multiple small aperture subarrays is proposed. First, the array output signals at different frequency bins are transformed into the focusing subspace, then the direction of arrival (DOA) are solved by a modified minimum variance estimator, the method does not need the accurate position information among each subarray, it handles the output information of multiple subarrays synthetically, consequently the error caused by position perturbation is diminished, and it has a preferable robustness to gain and phase intersubarray distortions, the performance has been proved by simulation results at last.*

Keywords: Direction of arrival(DOA) estimation, Wideband signals, Partly calibrated arrays

1. Introduction.

Super-resolution direction finding is also known as DOA estimation, it is one of the important research aspects in array signal processing, It concentrates on estimating the directions of arrival of interesting signals with multiple sensors, the main purpose is to determine the positions of the signals as well as other spatial parameters. The scholars have proposed many methods, such as harmonic analysis method[1], the maximum entropy method[2-3], Capon minimum variance method(MVM)[4], but they all subject to the distribution of the signal, and only adapt to narrowband signals. Since the end of 1970s, the area of spatial spectrum estimation have emerged a large number of achievements, for instance, maximum likelihood (ML)[5-7], MUSIC[8] and ESPRIT[9-10], as well coherent signal method(CSM)[11] which are suitable for wideband signals, but nearly all these methods are based on the premise of the ideal array position.

DOA estimation has been commonly used in civil and military fields, we need to choose the proper array manifold, traditional array requires a larger caliber to enhance the performance of the algorithm, and a dense arrangement is also necessary, but both of them will increase the complexity of system. In order to solve the problem, we can use intensive multiple small aperture arrays[12-17] to avoid fuzzy, meanwhile, multiple sparse large caliber arrays are employed to improve the estimation precision. In multiple array jointly estimation, on one hand, the distances among the subarrays usually do not satisfy the assumption of half wavelength, sometimes they are even up to a dozen wavelength, which will easily lead to the position error of the array sensors, so there are usually many false peaks when conventional super resolution direction finding methods are used; on the other hand, the gain and phase offset among these subarrays are often disaccord, which will cause the calculation failure. Thus the performance of DOA estimation will reduce sharply, and they have hindered the application of these methods in the actual system.

The paper proposed a new method for wideband signals based on partly calibrated multiple small aperture subarrays, it decomposes the output information of these subarrays at every frequency, then a modified minimum variance estimator is employed for DOA estimation, multiple subarrays jointly estimation extends the array aperture, improves the accuracy which is difficult to meet by the single small aperture array. Every single subarray can be calibrated offline in advance, the array signal model does not contain any position information among these subarrays, so we do not need to estimate the manifold of the whole array, and the modified estimator decreases the complex work brought by the array estimation and influence of the position error to the final result.

The rest of this paper is organized as follows: In the next section, we present the signal model and problem statement. In section 3 MVM estimator. In section 4 Proposed method. In section 5 Computational complexity. In section 6, Analysis of the experimental results. At last, we conclude in section 7.

2. **Signal model and problem statement.** Consider an array with K small aperture subarrays, each is composed of M_k sensors, so we have $M = \sum_{k=1}^{K} M_k$ sensors in all, there are N far-field wideband signals arriving at the array from $(\theta_1, \cdots, \theta_N)$ with the same power, they are zero mean stationary random, the noise background is Gaussian white process, and the signals and the noise are statistical independence.

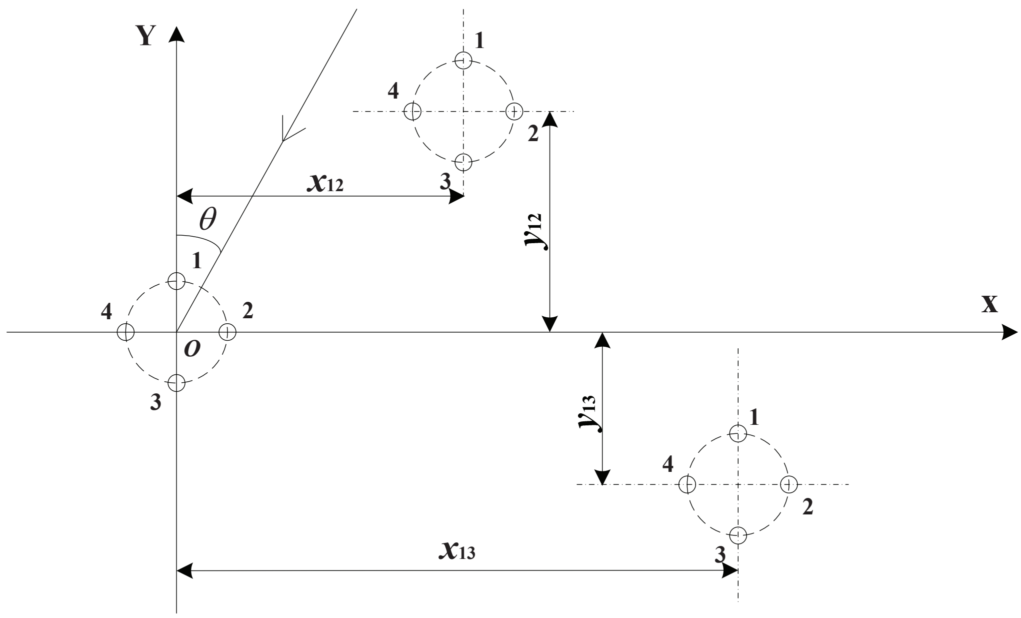

Figure 1. The structure of array

The array manifold at center frequency f can be expressed as

$$\boldsymbol{A} = [\boldsymbol{a}(f, \theta_1, \alpha), \boldsymbol{a}(f, \theta_2, \alpha), \cdots, \boldsymbol{a}(f, \theta_N, \alpha)] \tag{1}$$

Where $\boldsymbol{a}(f,\theta_n,\alpha)(n=1,2,\cdots,N)$ is a M dimensional scanning vector, and define α as the positional relation of array manifold among the subarrays, the scanning vector of the array is

$$\boldsymbol{a}(f,\theta,\alpha)=\boldsymbol{V}\boldsymbol{h} \tag{2}$$

Where

$$\boldsymbol{V}=\begin{bmatrix} \boldsymbol{V}_1\cdots\ 0\cdots\ 0 \\ 0\cdots\ \boldsymbol{V}_2\cdots\ 0 \\ \vdots \\ 0\cdots\ 0\cdots\ \boldsymbol{V}_K \end{bmatrix} \tag{3}$$

is a $M\times K$ dimensional scanning matrix, composing by scanning vector $\boldsymbol{V}_k(k=1,2,\cdots,K)$. If the first sensor of every small aperture subarray are taken as the reference point, then

$$\boldsymbol{V}_k=\begin{bmatrix} 1 \\ \exp[\mathrm{j}2\pi/\lambda(\xi_{2,k}\sin\theta+\zeta_{2,k}\cos\theta)] \\ \vdots \\ \exp[\mathrm{j}2\pi/\lambda(\xi_{M_k,k}\sin\theta+\zeta_{M_k,k}\cos\theta)] \end{bmatrix} \tag{4}$$

Where λ corresponds the wavelength of the center frequency f, $\xi_{m,k}$ and $\zeta_{m,k}$ are respectively the coordinates of the mth $(m=1,2,\cdots,M_k)$ sensor in the kth subarray with reference to the first sensor of the kth subarray, and $\boldsymbol{h}$ is a K dimensional complex vector, it expresses the information of array manifold among the subarrays, take the first subarray as the reference, then

$$\boldsymbol{h}=[h_1,h_2,\cdots,h_K]^{\mathrm{T}} \tag{5}$$

Clearly, h_1=1 and the other $h_k(k=1,2,\cdots,K)$ can be modeled as

$$h_k=g_k\exp(\mathrm{j}\varphi_k) \tag{6}$$

Where g_k and φ_k are respectively the unknown gain and phase mismatches between the kth and the first subarray.

If we consider the position error information among these subarrays, then $\boldsymbol{h}$ can be expressed as the Hadamard product of estimated value with error

$$\boldsymbol{h}=\hat{\boldsymbol{h}}\odot\boldsymbol{e} \tag{7}$$

Where $\odot$ is the Hadamard product; $\hat{\boldsymbol{h}}$ is the estimated value of $\boldsymbol{h}$, $\boldsymbol{e}$ is the vector caused by positional error, then the scanning vector can be defined as

$$\hat{\boldsymbol{a}}(f,\theta,\alpha)=\boldsymbol{V}\hat{\boldsymbol{h}} \tag{8}$$

3. **MVM estimator.** Consider an array consisting of M sensors, the receiving narrowband signal vector is $\boldsymbol{x}(t)$, the weighting vector of each channel is

$$\boldsymbol{w}=[w_1,w_2,\cdots,w_M]^{\mathrm{T}} \tag{9}$$

The array output is represented by

$$y(t)=\boldsymbol{w}^{\mathrm{H}}\boldsymbol{x}(t)=\sum_{i=1}^{M}{w^*}_i x_i(t) \tag{10}$$

Average power of the whole array is

$$P(\boldsymbol{w})=1/L\sum_{t=1}^{L}|y(t)|^2=\boldsymbol{w}^{\mathrm{H}}E[\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)]\boldsymbol{w}=\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}\boldsymbol{w} \tag{11}$$

Where L is the number of snapshots, our purpose is to guarantee to receive the actual signal we want correctly, while the interferences and signals from other directions are suppressed completely, that is

$$\begin{cases} \min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}} \boldsymbol{R} \boldsymbol{w} \\ \boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{a}}(\theta_d) = 1 \end{cases} \tag{12}$$

The corresponding optimal weighting vector can be deduced

$$\boldsymbol{w}_{\mathrm{opt}} = \frac{\boldsymbol{R}^{-1}\hat{\boldsymbol{a}}(\theta_d)}{\hat{\boldsymbol{a}}^{\mathrm{H}}(\theta_d)\boldsymbol{R}^{-1}\hat{\boldsymbol{a}}(\theta_d)} \tag{13}$$

So the spatial spectrum can be acquired by scanning the following function

$$P(\theta) = \frac{1}{\hat{\boldsymbol{a}}^{\mathrm{H}}(\theta)\boldsymbol{R}^{-1}\hat{\boldsymbol{a}}(\theta)} \tag{14}$$

4. **Proposed method.** The algorithm for wideband signals can be easily deduced by the results above, it is summarized as two steps: wideband focusing and modified MVM estimator, they are described below.

4.1. **Wideband focusing.** First, focusing is employed, signals of all frequency bins are transformed into the focusing subspace, then the steady estimator is founded to calculate DOA, wideband signal in frequency domain is shown as

$$\boldsymbol{X}(f_i) = \boldsymbol{A}(f_i, \theta, \alpha)\boldsymbol{S}(f_i) + \boldsymbol{N}(f_i) \ (i = 1, 2, \cdots, J) \tag{15}$$

Where $\boldsymbol{A}(f_i, \theta, \alpha) = [\boldsymbol{a}(f_i, \theta_1, \alpha), \boldsymbol{a}(f_i, \theta_2, \alpha), \cdots, \boldsymbol{a}(f_i, \theta_N, \alpha)]$ is a $M \times N$ dimensional direction matrix; $\boldsymbol{S}(f_i)$ is a $N \times 1$ dimensional signal vector; $\boldsymbol{N}(f_i)$ is a $M \times 1$ dimensional noise vector, J is the number of frequency bins, the covariance matrix is

$$\boldsymbol{R}_{XX}(f_i) = E[\boldsymbol{X}(f_i)\boldsymbol{X}^{\mathrm{H}}(f_i)] = \boldsymbol{A}(f_i)\boldsymbol{R}_{SS}(f_i)\boldsymbol{A}^{\mathrm{H}}(f_i) + \sigma^2\boldsymbol{I} \ (i = 1, 2, \cdots, J) \tag{16}$$

Then we can use some focusing method to acquire the average matrix $\hat{\boldsymbol{R}}_Y(f_0)$ by transforming the signals of different frequency bins into focusing subspace.

4.2. **Modified MVM estimator.** Here, two errors are mainly considered, one is amplify intersubarray distortions, it leads to the gain disaccord among the subarrays; the other is phase intersubarray distortions, it leads to the phase offset disaccord among the subarrays, we will discuss them respectively below.

4.2.1. *Amplify intersubarray distortions.* A modified MVM estimator can be expressed by[16-17]

$$P_{BL}(\theta) = \frac{\hat{\boldsymbol{a}}^{\mathrm{H}}(\theta)\hat{\boldsymbol{R}}_Y^{-1}\hat{\boldsymbol{a}}(\theta)}{\hat{\boldsymbol{a}}^{\mathrm{H}}(\theta)\hat{\boldsymbol{R}}_Y^{-2}\hat{\boldsymbol{a}}(\theta)} \tag{17}$$

It can be expressed as another form

$$P_{BL}(\theta) = \frac{\hat{\boldsymbol{h}}^{\mathrm{H}}\boldsymbol{B}_1(\theta)\hat{\boldsymbol{h}}}{\hat{\boldsymbol{h}}^{\mathrm{H}}\boldsymbol{B}_2(\theta)\hat{\boldsymbol{h}}} \tag{18}$$

Where

$$\boldsymbol{B}_1(\theta) = \boldsymbol{V}^{\mathrm{H}}(\theta)\hat{\boldsymbol{R}}_Y^{-1}\boldsymbol{V}(\theta) \tag{19}$$

$$\boldsymbol{B}_2(\theta) = \boldsymbol{V}^{\mathrm{H}}(\theta)\hat{\boldsymbol{R}}_Y^{-2}\boldsymbol{V}(\theta) \tag{20}$$

Here, $\hat{\boldsymbol{a}}(\theta)$, $\hat{\boldsymbol{h}}$, $\hat{\boldsymbol{R}}_Y$ and $\boldsymbol{V}(\theta)$ are all used the vectors which are at the focusing frequency, thus, the estimator can be extended to wideband signals.

Here, an amplify intersubarray distortions is considered, a worst-case optimization problem is

$$\max_{\boldsymbol{h}} P_{BL}(\theta) \tag{21}$$

Equation (21) can be deemed to be a convex quadratic optimization problem and it is expressed

$$\max_{\boldsymbol{h}} \hat{\boldsymbol{h}}^{\mathrm{H}}\boldsymbol{B}_1(\theta)\hat{\boldsymbol{h}} \tag{22}$$

s.t.

$$\hat{\boldsymbol{h}}^{\mathrm{H}}\boldsymbol{B}_2(\theta)\hat{\boldsymbol{h}}=1$$

It can be solved by Lagrange function

$$f(\boldsymbol{h}) = \hat{\boldsymbol{h}}^{\mathrm{H}}\boldsymbol{B}_1(\theta)\hat{\boldsymbol{h}} - \mu(\hat{\boldsymbol{h}}^{\mathrm{H}}\boldsymbol{B}_2(\theta)\hat{\boldsymbol{h}} - 1) \tag{23}$$

By calculating its extreme value, the following equation can be acquired

$$\boldsymbol{B}_2^{-1}(\theta)\boldsymbol{B}_1(\theta)\hat{\boldsymbol{h}}(\theta) = \mu\hat{\boldsymbol{h}}(\theta) \tag{24}$$

So the solution of (22) is

$$\hat{\boldsymbol{h}}(\theta) = \eta\ell_{\max}[\boldsymbol{B}_2^{-1}(\theta)\boldsymbol{B}_1(\theta)] \tag{25}$$

Where $\ell_{\max}[\cdot]$ returns the eigenvector corresponding to the maximum eigenvalue of the matrix $\boldsymbol{B}_2^{-1}(\theta)\boldsymbol{B}_1(\theta)$. We can see that $\hat{\boldsymbol{h}}(\theta)$ is equal to the eigenvector of $\boldsymbol{B}_2^{-1}(\theta)\boldsymbol{B}_1(\theta)$ with η, which provided to perform according to (25) by referencing to (22) with the constraint

$$\hat{\boldsymbol{h}}^{\mathrm{H}}(\theta)\boldsymbol{B}_2(\theta)\hat{\boldsymbol{h}}(\theta) = 1 \tag{26}$$

We can see from (24)

$$\hat{\boldsymbol{h}}^{\mathrm{H}}(\theta)\boldsymbol{B}_1(\theta)\hat{\boldsymbol{h}}(\theta) = \mu \tag{27}$$

So the spectrum of the estimator is equal to the maximum eigenvalue of multiplier of the two matrice and can be described as

$$P_{RBL}(\theta) = \dagger_{\max}[\boldsymbol{B}_2^{-1}(\theta)\boldsymbol{B}_1(\theta)] \tag{28}$$

Where $\dagger_{\max}[\ ]$ is the symbol returning the maximum eigenvalue of the matrix.

4.2.2. *Phase Intersubarray Distortions.* In most cases, the wave front aberration is mainly affected by random or unknown deterministic perturbations, so the amplitude distortions can be ignored, it can be seen from (6), the phase distortions satisfy

$$|h_k| = 1, k = 1, 2, \cdots, K \tag{29}$$

Thus, the vector $\boldsymbol{h}$ based on worst-case optimization is

$$\min_{\boldsymbol{h}} \hat{\boldsymbol{h}}^{\mathrm{H}}\boldsymbol{B}_1(\theta)\hat{\boldsymbol{h}} \tag{30}$$

s.t.

$$|h_k| = 1, k = 1, 2, \cdots, K$$

A more accurate solution[19] based on semidefinite programming (SDP) algorithm can be used here for the solution, it is described as follows:

$$\max_{\boldsymbol{h}} \mathrm{Tr}[\boldsymbol{B}_1(\theta)\boldsymbol{H}] \tag{31}$$

s.t.

$$\mathrm{Tr}[\boldsymbol{B}_2(\theta)\boldsymbol{H}] = 1, \{\boldsymbol{H}\}_{k,k} = 1, k = 1, 2, \cdots, K.\ \boldsymbol{H} \succ 0, \mathrm{rank}[\boldsymbol{H}]=1.$$

Where Tr[·] is the trace operator, $\{\cdot\}_{k,k}$ shows the kth diagonal element of the matrix, $\boldsymbol{H} \succ 0$ denotes that $\boldsymbol{H}$ is positive semidefinite.

Obviously, equation (31) is a non-convex problem, the non-convex rank constraint rank$[\boldsymbol{H}] = 1$, it is necessary to change it to be convex, here, the semidefinite relaxation algorithm[20] is employed to solve the problem, thus, non-convex rank constraint is dropped, it is

$$\max_{\boldsymbol{h}} \mathrm{Tr}[\boldsymbol{B}_1(\theta)\boldsymbol{H}] \tag{32}$$

s.t.

$$\mathrm{Tr}[\boldsymbol{B}_2(\theta)\boldsymbol{H}] = 1, \{\boldsymbol{H}\}_{k,k} = 1, k = 1, 2, \cdots, K.\ \boldsymbol{H} \succ 0.$$

In this way, the spectrum can be solved by (32), RARE[14] and RC[21] algorithms can also be extended to the DOA estimation for wideband signals with the similar way, so they can be defined as WRARE and WRC method respectively.

5. **Computational complexity.** It is necessary to analyze computational complexity of the method, define the number of the sensors of the array is M, and the number of sampling frequency bins is J, that of snapshots of every frequency is L, the complexity of the traditional CSM method concentrates on the focusing and spectrum searching, so the calculation of focusing is $O(M^3LJ)$, that of the spectrum searching is $O(M^2LJ + M^3)$, so its total calculation is $O(M^3LJ + M^2LJ + M^3)$; the proposed method based on (28) contains focusing for wideband signals, solution for $\boldsymbol{B}_1(\theta)$ and $\boldsymbol{B}_2(\theta)$, the calculation of focusing is equal to $O(M^3LJ)$ too, and that of the matrix $\hat{\boldsymbol{R}}_Y$ inversion is $O(M^3)$, the calculation for $\boldsymbol{B}_1(\theta)$ and $\boldsymbol{B}_2(\theta)$ is equal to $O(M^3L)$, so its total calculation is $O(M^3(LJ + L + 1))$ approximately; The method based on SDP algorithm is shown as (32), the complexity of the equation is analogously as $O(M^{3.5}L)$, so its total calculation is $O(L(M^3J + M^{3.5}))$, the proposed method is based on modified MVM estimator for wideband signals, so it can be called WMVM for short.

6. **Analysis of the experimental results.** In the simulations, assume some wideband acoustic signals arriving at the array, it is seen from the derivation above, the course of focusing can solve the coherence among the signals, so there is no special limitation for the correlation among signals, here assume they are all uncorrelated, the array is composed of three same circle identically oriented subarrays, each subarray has four sensors evenly distributing on the circumference, their radiuses are $\lambda/4 = 0.5$m, where λ is the wavelength corresponding to the frequency 170Hz, bandwidth $B = 40$Hz, the background noise is uncorrelated with the signal, it is a white Gaussian stationary random process with zero means and statistically independent on each sensor. We can acquire position information among the subarrays by manifold estimation and correction, here define position of the second subarray relative to the first one is $[10\lambda, 6\lambda]$, and that of the third subarray relative to the first one is $[20\lambda, -5\lambda]$, the structure is shown in Figure.1.

The sampling frequency of simulation is taken as 600Hz, the observation time is 16s, it is divided into 32 periods, the whole frequency band is divided into 31 subbands through Fast Fourier Transform (FFT), in order to reduce the spectrum leakage, Hanning window is used for FFT, overlapped data is at a rate of 20%, the WRARE, WRC and WMVM methods are employed below for the performance comparison of DOA estimation.

Simulation 1 Precision of the methods versus SNR

A. Amplify intersubarray distortions

Consider three far-field wideband signals arriving at the sensors from 18°, 26° and 35° with the same power, here we test the performance of these methods with amplify intersubarray distortions versus SNR, phase shift ratios of the array sensors are the same, the gain ratios of three subarrays are respectively 0.8, 0.9 and 1.2, they reflect the different weightings of every subarray, snapshots in every frequency is 50, SNR varies from -5dB to 15dB, step size is 1dB, the estimated positions of the second and the third small aperture subarrays respectively deviate from the actual location $[-0.36\lambda, 0.18\lambda]$ and $[0.33\lambda, 0.35\lambda]$, 400 times Monte Carlo trials have run for each SNR, the average of them is regarded as the measured result for this SNR, Figure.2 shows the RMSE of WRARE, WRC and WMVM methods based on (28) versus SNR.

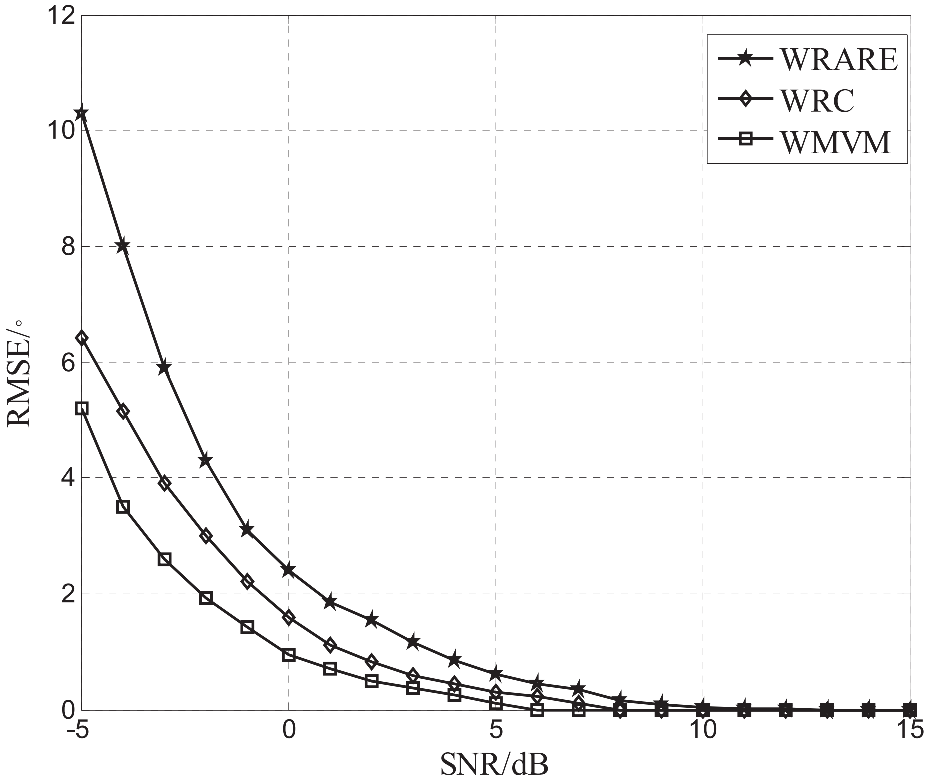

FIGURE 2. RMSE of the methods versus SNR(amplify intersubarray distortions)

From Figure.2 it is seen that RMSE of WMVM is smaller than that of the other two methods, when SNR reaches 6dB, RMSE of WMVM is zero, but that of WRC and WRARE methods reach zero when SNR are respectively 8dB and 10dB.

B. Phase intersubarray distortions

Consider three far-field wideband signals arriving at the sensors from 18°, 26° and 35° with the same power, here we test the performance of these methods with phase intersubarray distortions versus SNR, the gain ratios of the array sensors are the same, phase shift ratios of the array sensors are 0°, 9° and 7°, it reflects the different phase offsets of every subarray, snapshots in every frequency is 50, SNR varies from -5dB to 15dB, step size is 1dB, the estimated positions of the second and the third small aperture subarrays respectively deviate from the actual location $[-0.36\lambda, 0.18\lambda]$ and $[0.33\lambda, 0.35\lambda]$, 400 times Monte Carlo trials have run for each SNR, the average of them is regarded as the measured result for this SNR, Figure.3 shows the RMSE of WRARE, WRC and WMVM methods based on (32) versus SNR.

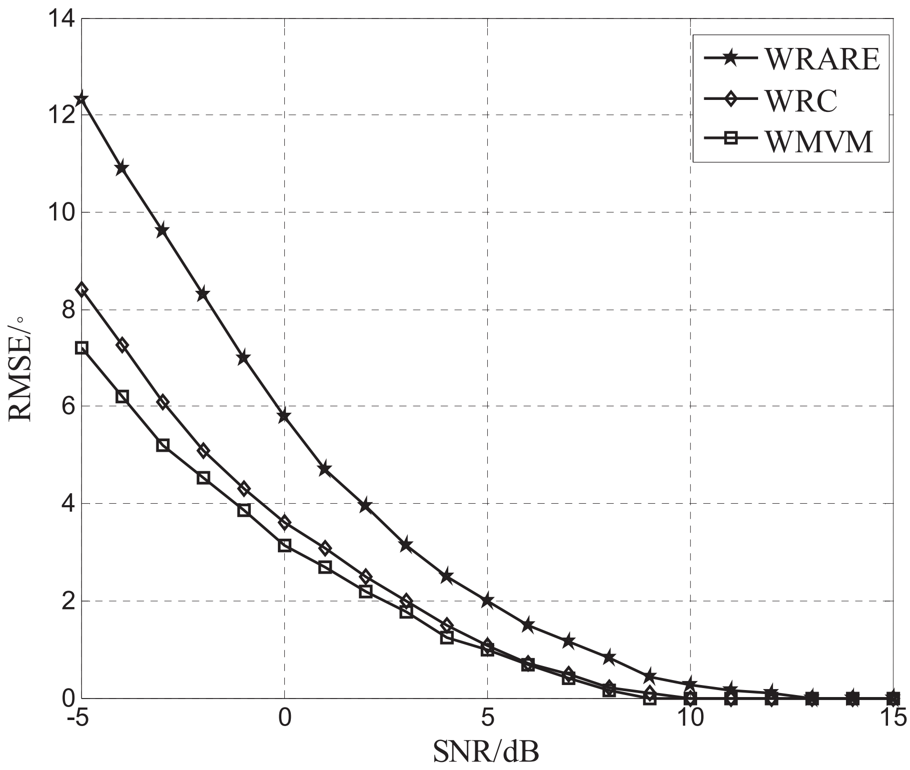

Figure 3. RMSE of the methods versus SNR(phase intersubarray distortions)

From Figure.3 it is seen that RMSE of WMVM is smaller than that of the other two methods, when SNR reaches 9dB, RMSE of WMVM is zero, but that of WRC and WRARE methods reach zero when SNR are respectively 10dB and 13dB.

Simulation 2 Precision of the methods versus snapshots

A. Amplify intersubarray distortions

Consider three far-field wideband signals arriving at the sensors from 18°, 26° and 35° with the same power, here we test the performance of these methods with phase intersubarray distortions versus snapshots, phase shift ratios of the array sensors are the same, the gain ratios of three subarrays are respectively 0.8, 0.9 and 1.2, they reflect the different weightings of every subarray, SNR is 15dB, snapshots varies from 5 to 50 in every frequency, step size is 5, position error is the same with simulation 1, 400 times Monte Carlo trials have run for each snapshots, the average of them is regarded as the measured result for this snapshots, Figure.4 shows the RMSE of WRARE, WRC and WMVM methods based on (28) versus snapshots.

From Figure.4 it is seen that RMSE of WMVM is smaller than that of the other two methods, when snapshots reaches 30, RMSE of WMVM is zero, but that of WRC and WRARE methods reach zero when snapshots are respectively 35 and 40.

B. Phase intersubarray distortions

Consider three far-field wideband signals arriving at the sensors from 18°, 26° and 35° with the same power, here we test the performance of these methods with phase intersubarray distortions versus snapshots, the gain ratios of the array sensors are the same, phase shift ratios of the array sensors are 0°, 9° and 7°, it reflects the different phase offsets of every subarray, SNR is 15dB, snapshots varies from 5 to 50 in every frequency, step size is 5, position error is the same with simulation 1, 400 times Monte Carlo trials have run for each snapshots, the average of them is regarded as the measured

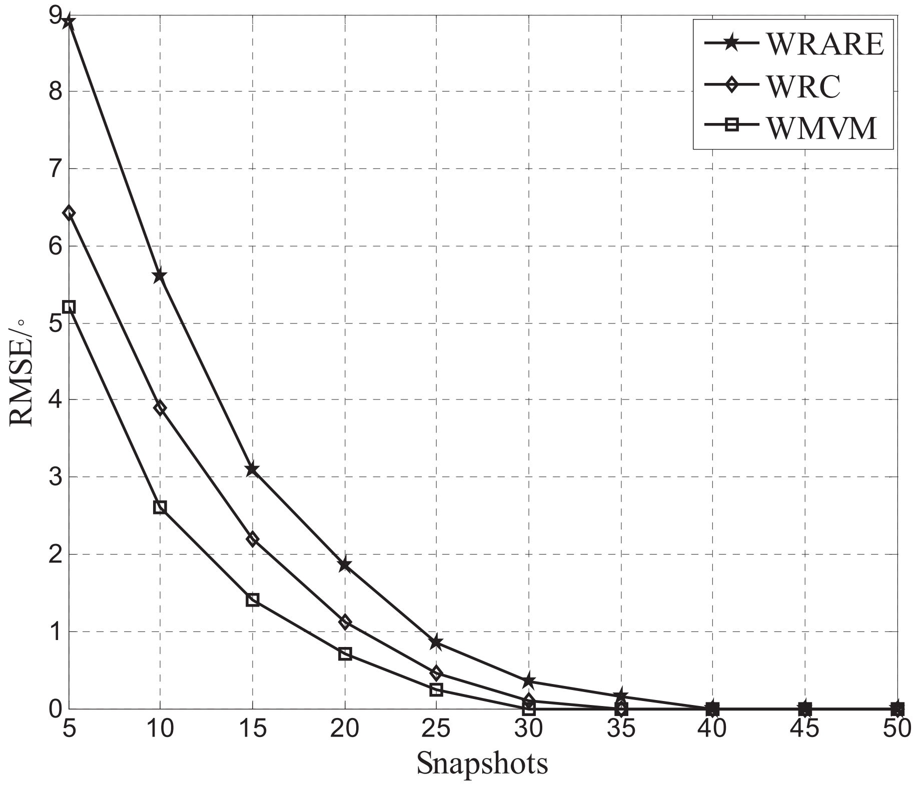

Figure 4. RMSE of the methods versus snapshots(amplify intersubarray distortion)

result for this snapshots, Figure.5 shows the RMSE of WRARE, WRC and WMVM methods based on (32) versus snapshots.

From Figure.5 it is seen that RMSE of WMVM is smaller than that of the other two methods, when snapshots reaches 35, RMSE of WMVM is zero, but that of WRC and WRARE methods reach zero when snapshots are respectively 40 and 45.

7. **Conclusion.** A novel super-resolution direction finding method for wideband signals based on multiple small aperture subarrays has been proposed in the paper, it uses modified minimum variance estimator to handle the output information of multiple subarrays synthetically, decreases the error caused by sensor perturbation, or discordance of gain and phase among subarrays effectively, and it is more preferable than WRARE and WRC methods in the same conditions.

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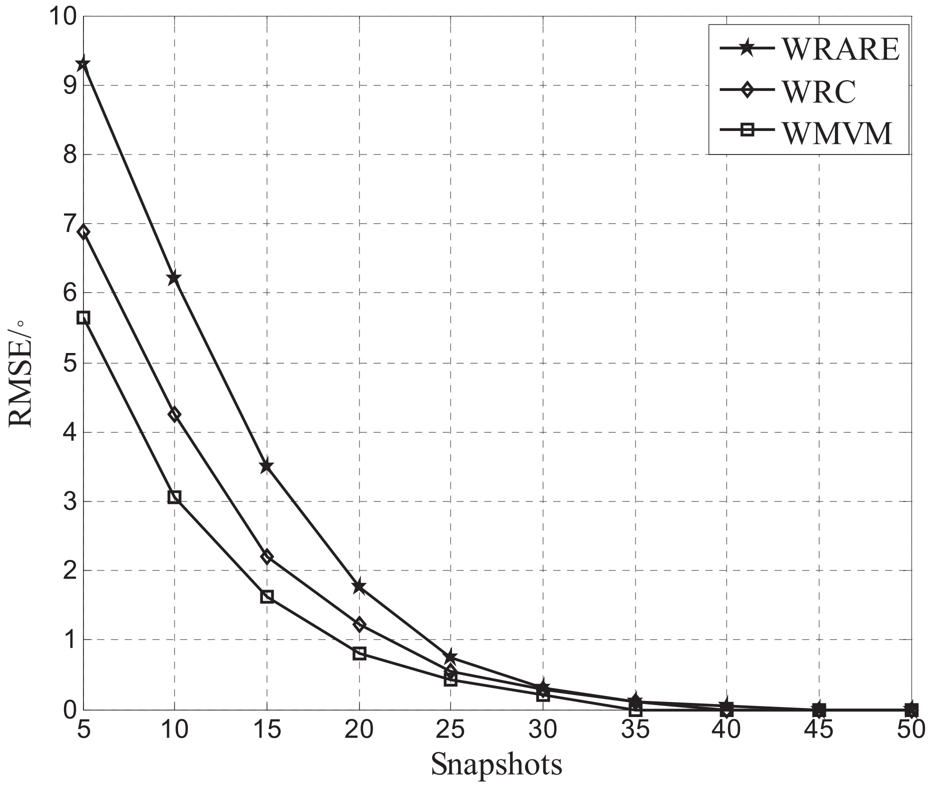

FIGURE 5. RMSE of the methods versus snapshots(phase intersubarray distortions)

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