## Invariability of Mean Value Based Reversible Watermarking

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ABSTRACT. By employing invariant relation between the mean value of the first (n - 1)1) pixels and the last one pixel (also called the remaining pixel) for every image block containing n pixels, a new reversible watermark scheme capable of mostly carrying 2n-3bits into one n-sized image block in a single embedding process is presented in this paper. First, the mean value of the first (n-1) pixel is calculated. Next, the difference value between the last one pixel and this obtained mean value is applied to distinguish which classification (i.e., smooth or complex sub-block) any sub-block belongs to. Consequently, it is determined to embed (n-2) bits or 2(n-2) bits into each sub-block according to its final classification results. And meanwhile, the mean value is reapplied to predict this last one pixel. 1-bit watermark is embedded into this last one pixel in accordance with the magnitude of prediction-error value. By multi-employing invariability of the mean value of (n-1) pixels, the embedding rate can approach to  $(2-\frac{3}{n})$  bpp (bit per pixel) for a single embedding process. Meanwhile, the embedding distortion is greatly controlled by embedding more bits into smooth image blocks and fewer bits into the other blocks with complex texture. Experimental results reveal the proposed scheme is effective. Keywords: Reversible Watermarking, Invariability of Mean Value

1. Introduction. Reversible watermarking based on difference expansion was proposed by Tian [1]. The differences between two pixels were expanded to carry watermark information if neither overflow nor underflow occurred. Alattar [2] generalized the DE technique by taking a set containing multiple pixels rather than a pair. Coltuc *et al.* propose a threshold-controlled embedding scheme based on an integer transform for pairs of pixels [3]. In Thodi's work [4], histogram shifting was incorporated into Tian's method to produce a new algorithm called Alg. D2 with a overflow map. Although Alg. D2 has achieved high performance relative to Tian's method, its embedding rates still can not exceed 0.5 bpp (bit per pixel) for a single embedding processing. According to the performance comparison figure in paper [4], a inflexion point appeared at around 0.5 when multiple embedding was applied to achieve rates close to 1 bpp. Weng *et al.* [5] proposed an integer transform based on invariability of the sum of pixel pairs [5]. Though Weng *et al.*'s method can achieve embedding rates of about 1 bpp for a single embedding process, it is not capable of embedding small payloads at low distortions for some medical images. Weng *et al.*'s method [6] proposed a new integer transform, and the embedding rate can approach to 1 bpp for a single embedding process by overlapped pairing. In Wang *et al.*'s method [7], an generalized integer transform and a payload-dependent location map were constructed to extend the DE technique to the pixel blocks of arbitrary length.

In Wang *et al.*'s method, the variation of any image block is proposed to determine whether this block can be used for embedding or not. The variation is only a coarse estimation for correlation among pixels, which can not reflect on the detailed relationship between each pixel and its mean value. With the threshold increased, sub-blocks with weak inter-pixel correlation are also possibly classified into the embeddable sub-block, and hence, the distortion introduced by modifying these sub-blocks also is higher. In the simulate study for Wang *et al.* 's method, it is found that the distortion has a drastic increment as the embedding rate increases.

To solve this problem above, in the proposed method, all image blocks are classified into one of two categories (namely smooth and complex (also called high-textured) sub-block) according to inter-pixel correlation strength. The strength of correlation is measured depending on the relation between the mean value of the first (n-1) pixels and the last one pixel. Different embedding strategy is adopted aiming at different types of blocks. In other words, more bits are embedded into smooth blocks while fewer in high-textured blocks. Histogram shifting technique is applied to the proposed method so as to more efficiently compress the location map which is used for recording the overflow or underflow positions. In fact, the introduction of histogram shifting is bound to increase distortion while have no help to an increase in payload. Therefore, at low embedding rates, the distortion introduced by histogram shifting itself is more obvious than that created by the process of embedding watermark. As a result, Wang et al.'s method performs better than the proposed method at low embedding rates. With the embedding bit-rates increased, the number of sub-blocks used for histogram shifting is decreased, and hence, the distortion introduced by histogram shifting is decreased. As a result, the proposed method is significantly better than Wang et al. 's method at high embedding rates.

2. Wang's method. In Wang's *et al.*'s method, a generalized integer transform based on invariability of the variation of an image block is proposed.

$$y_{1} = 2x_{1} - 2f(a(\mathbf{x})) + w_{1}$$
  
...  

$$y_{n-1} = 2x_{n-1} - 2f(a(\mathbf{x})) + w_{n-1}$$
  

$$y_{n} = 2x_{n} - a(\mathbf{x})$$
(1)

The integer transform defined in Wang et al.'s method is modified as follows.

$$y_{1} = 2x_{1} - (a(\mathbf{x}) + LSB(a(\mathbf{x}))) + w_{1}$$
  
...  

$$y_{n-1} = 2x_{n-1} - (a(\mathbf{x}) + LSB(a(\mathbf{x}))) + w_{n-1}$$
  

$$y_{n} = 2x_{n} - a(\mathbf{x})$$
(2)

where **x** is a pixel block containing *n* pixels, i.e.,  $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{Z}^n$ ,  $a(\mathbf{x}) = \begin{cases} \lfloor \bar{\mathbf{x}} \rfloor & \text{if } \bar{\mathbf{x}} - \lfloor \bar{\mathbf{x}} \rfloor < 0.5 \\ \lceil \bar{\mathbf{x}} \rceil & \text{otherwise} \end{cases}$ ,  $\bar{\mathbf{x}} = \frac{1}{n} \sum_{i=1}^n x_i$ ,  $f(\cdot) = \lfloor \frac{\mathbf{x}}{2} \rfloor = \frac{\mathbf{x} + LSB(\mathbf{x})}{2}$ ,  $LSB(\cdot)$  represents the least significant bit (LSB).  $a(\mathbf{x})$  is actually the rounded value of  $\bar{\mathbf{x}}$ .  $\mathbf{y} = (y_1, \dots, y_n)$  is used to denote the corresponding watermarked pixel block of  $\mathbf{x}$ .



FIGURE 1. The corresponding partition way for a  $2 \times 2$  and  $3 \times 3$  images block, where (n-1) pixels of any block are marked as black-filled circles, while the remaining pixel marked as the blue-filled circle

In Wang et al.'s method, the variation of any image block is proposed to determine whether this block can be used for embedding or not. Take an image block  $\mathbf{x_1} = (13, 5, 6)$ for example, its mean value, i.e.,  $a(\mathbf{x_1})$ , is 8, and its variation value denoted by  $v(\mathbf{x_1})$  is calculated using the following formula:  $\sqrt{(x_1 - a(\mathbf{x_1}))^2 + (x_2 - a(\mathbf{x_1}))^2 + (x_2 - a(\mathbf{x_1}))^2} =$  $\sqrt{38}$ . If 2-bit watermark is set to '10', then  $\mathbf{y_1} = (19, 2, 4)$  after watermark embedding. For another image block  $\mathbf{x_2} = (13, 6, 8)$ , its  $a(\mathbf{x_2})$  is 9, and its the variation is  $\sqrt{26}$ . After 2-bit watermark '10' are embedded into  $\mathbf{x_2}$ ,  $\mathbf{y_2} = (18, 3, 7)$ . For  $\mathbf{x_1}$ , its (n-1) difference values which are calculated by subtracting the grayscale value of each pixel from its mean value are 5, -3, and -2 respectively. Similarly, for  $\mathbf{x}_2$ , the difference values are 4, -3, and -1, respectively. Here, let the given threshold denoted by the notation  $T_h$  be 7. Since  $0 \leq \mathbf{y_i} \leq 255$ , and  $|v(\mathbf{x}_i)| \leq T_h$ , two bits must be embedded into block  $\mathbf{x_i}$  despite the fact that some difference value is a larger value, e.g., 5, 4 and -3, where  $1 \leq i \leq 2$  and  $i \in \mathbb{Z}$ , and  $|\cdot|$  represents the absolute value. It can be seen from the example above that the variation is only a coarse estimation for correlation among pixels, which can not reflect on the detailed relationship between each pixel and its mean value. With the threshold increased, sub-blocks with weak inter-pixel correlation are also possibly changed into the embeddable sub-block, and hence, the distortion introduced by modifying these subblocks also is higher. In the simulate study for Wang *et al.* 's method, it is found that the distortion has a drastic increment as the embedding rate increases.

To solve the problem above, in the proposed method, all the image blocks will firstly be classified into smooth and complex blocks according to the correlation between the mean value of the first (n-1) pixels and the last one pixel. In accordance with the classification results of sub-blocks, the correlation among pixels must be reconsidered, if the correlation is strong, 2-bit watermark is embedded into this difference values of smooth blocks, and meanwhile, one bit is embedded into this difference value of complex blocks. Hence, the embedding distortion is greatly decreased by embedding more bits into smooth image blocks while embedding fewer bits into blocks with complex texture.

3. The proposed method. In the proposed method, a grayscale image is partitioned into non-overlapping *n*-sized sub-blocks, where  $n = m \times m$ . Any  $m \times m$ -sized sub-block  $\mathbf{x}_n$  is arranged into a dimensional list based on high-correlation existing between two neighboring pixels, i.e.,  $\mathbf{x}_n = (x_1, x_2, \dots, x_n) \in \mathbb{Z}^n$ , its first (n-1) pixels are grouped into a sub-set denoted by  $\mathbf{x}_{n-1}$ .

Take a 2 × 2 sub-block for example, the bottom-right pixel  $x_4$  marked as blue-filled circle is selected as the remaining pixel, the other three pixels are selected to form sub-set  $\mathbf{x_{n-1}}$  as illustrated in Fig. 1(a). The center pixel  $x_9$  of a 3 × 3 image block is selected

to be this remaining pixel, the other eight pixels surrounding  $x_9$  shown in Fig. 1(b) are selected to form sub-set  $\mathbf{x_{n-1}}$ .

Correspondingly, the embedding idea is also partitioned into two parts. The first one is the process of obtaining the detailed classification of  $\mathbf{x_{n-1}}$  and then producing the corresponding embedding strategy for  $\mathbf{x_{n-1}}$ . The other one is the process of using the mean value, denoted by the notation  $\mathbf{\bar{x}_{n-1}}$  of the first (n-1) pixels to predict the remaining pixel  $x_n$ .

3.1. Process of producing classification and embedding strategy for each  $x_{n-1}$ . The mean value  $\bar{x}_{n-1}$  of sub-set  $x_{n-1}$  is calculated using the following formula.

$$\bar{\mathbf{x}}_{\mathbf{n-1}} = \lfloor \frac{x_1 + x_2 + \dots + x_{n-1}}{n-1} \rfloor \tag{3}$$

The relation between  $\bar{\mathbf{x}}_{n-1}$  and  $vT_h$  is used to estimate whether each sub-block has strong inter-pixels correlation or not. If  $|\bar{\mathbf{x}}_{n-1} - x_n| \leq vT_h$ , then this  $\mathbf{x}_{n-1}$  is regarded as smooth one, where  $vT_h$  is a predefined threshold which is used for distinguishing which classification sub-set  $\mathbf{x}_{n-1}$  belongs to. Otherwise,  $\mathbf{x}$  is a complex sub-set.

Alattar's idea based on non-variability of the mean value is applied to  $\mathbf{x_{n-1}}$  to create (n-2) difference values. The detailed integer transform  $T(\cdot)$  is described as follows.

$$\bar{\mathbf{x}}_{\mathbf{n-1}} = \left\lfloor \frac{x_1 + x_2 + \dots + x_{n-1}}{n-1} \right\rfloor \\
d_1 = x_2 - x_1 \\
d_2 = x_3 - x_2 \\
\dots \\
d_{n-2} = x_{n-1} - x_{n-2}$$
(4)

The inverse integer transform  $T^{-1}(\cdot)$  is defined as follows.

$$\begin{aligned}
x_1 &= \bar{\mathbf{x}}_{\mathbf{n}-1} - \left\lfloor \frac{(n-2) \times d_1 + (n-3) \times d_2 + \dots + 2d_{n-3} + d_{n-2}}{n-1} \right\rfloor \\
x_2 &= d_1 + x_1 \\
x_3 &= d_2 + x_2 \\
\dots \\
x_{n-1} &= d_{n-2} + x_{n-2}
\end{aligned} \tag{5}$$

For a smooth sub-set, if its difference value  $d_i$  falls into the range of  $[-pT_h, pT_h)$ ,  $d_i$  is expanded twice according to Eq. 6, where  $1 \le i \le (n-2)$  and  $i \in \mathbb{Z}$ ,  $pT_h$  is used for estimating the correlation between two neighboring pixels,  $e_i$  represents 2-bit watermark, i.e.,  $e_i \in \{0, 1, 2, 3\}$ .

$$d'_i = 4 \times d_i + e_i \tag{6}$$

If  $d_i \ge pT_h$  and  $d_i \le -pT_h - 1$ , it will be shifted according to the following formula

$$d'_{i} = \begin{cases} d_{i} - 3 \times pT_{h} & d \leq -pT_{h} - 1\\ d_{i} + 3 \times pT_{h}, & d \geq pT_{h} \end{cases}$$
(7)

For a complex sub-set, if its difference value  $d_i$  falls into the range of  $[-pT_h, pT_h)$ ,  $d_i$  is expanded only once based on Eq. 8, where  $1 \le i \le n-2$  and  $i \in \mathbb{Z}$ .

$$d'_i = 2 \times d_i + b_i \tag{8}$$

where  $b_i \in \{0, 1\}$ . If  $d_i \ge pT_h$  and  $d_i \le -pT_h - 1$ ,  $d_i$  will be shifted according to the following formula

$$d'_{i} = \begin{cases} d_{i} - pT_{h} & d \leq -pT_{h} - 1\\ d_{i} + pT_{h}, & d \geq pT_{h} \end{cases}$$

$$\tag{9}$$

3.2. Predicting pixel  $x_n$  using the mean value  $\bar{\mathbf{x}}_{n-1}$ . The pixel  $x_n$  is predicted using the mean value  $\bar{\mathbf{x}}_{n-1}$  to obtain the prediction-error. Assume  $d_n$  to be difference value between  $x_n$  and  $\bar{\mathbf{x}}_{n-1}$ , the embedding process is described as follows

$$y_{n} = \begin{cases} \bar{\mathbf{x}}_{n-1} + 2d_{n} + b_{n} & d_{n} \in [-pT_{h}, pT_{h}) \\ x_{n} - pT_{h} & d_{n} \leq -pT_{h} - 1 \\ x_{n} + pT_{h}, & d_{n} \geq pT_{h} \end{cases}$$
(10)

3.3. Comparison with Wang *et al.*'s method. Since  $\bar{\mathbf{x}}_{n-1} = \bar{\mathbf{y}}_{n-1}$ , substitute it and Eq. 8 into Eq. 5, we have

$$y_{1} = \bar{\mathbf{y}}_{n-1} - \left\lfloor \frac{d_{\Sigma} + b_{\Sigma}}{n-1} \right\rfloor$$

$$y_{2} = 2d'_{1} + y_{1}$$

$$y_{3} = 2d'_{2} + y_{2}$$

$$\cdots$$

$$y_{n-1} = 2d'_{n-2} + y_{n-2}$$
(11)

where  $d_{\Sigma} = 2((n-2) \times d_1 + (n-3) \times d_2 + \dots + 2d_{n-3} + d_{n-2}), b_{\Sigma} = (n-2) \times b_1 + (n-3) \times b_2 + \dots + b_{n-2}$ . After simplification,  $d_{\Sigma}$  is equal to  $2(\sum_{i=0}^{n-1} x_i - (n-1) \times x_1)$ . On the other hand, since  $b_i \in \{0,1\}, 1 \leq i \leq n-2$  and  $i \in \mathbb{Z}$ , then  $0 \leq b_{\Sigma} \leq \frac{(n-1) \times (n-2)}{2}$  and  $b_{\Sigma} \in \mathbb{Z}$ . Suppose  $x_1 + x_2 + \dots + x_{n-1}$  equals  $(n-1)k_1 + k_2$ , where  $k_1$  is actually  $\bar{\mathbf{x}}_{\mathbf{n-1}}, k_2 \in \{0, 1, \dots, n-2\}$ . Substitute this resulting into Eq. 11,  $y_1 = 2x_1 - \bar{\mathbf{x}}_{\mathbf{n-1}} - \lfloor \frac{b_{\Sigma} + 2k_2}{n-1} \rfloor$ ,  $y_2 = 2x_2 - \bar{\mathbf{x}}_{\mathbf{n-1}} - \lfloor \frac{b_{\Sigma} + 2k_2}{n-1} \rfloor + b_1, \dots, y_{n-1} = 2x_{n-1} - \bar{\mathbf{x}}_{\mathbf{n-1}} - \lfloor \frac{b_{\Sigma} + 2k_2}{n-1} \rfloor + \sum_{i=1}^{n-2} b_i$ , where  $0 \leq \lfloor \frac{b_{\Sigma} + 2k_2}{n-1} \rfloor \leq \lfloor \frac{(n-2)(n+3)}{2(n-1)} \rfloor$ , and  $\lfloor \frac{b_{\Sigma} + 2k_2}{n-1} \rfloor \in \mathbb{Z}$ . The formula  $\lfloor \frac{(n-2)(n+3)}{2(n-1)} \rfloor$  itself seems too complex, and hence, the notation  $d_e$  is created to denote it. As a result,  $2x_1 - \bar{\mathbf{x}}_{\mathbf{n-1}} - d_e \leq y_1 \leq 2x_1 - \bar{\mathbf{x}}_{\mathbf{n-1}}, \dots$ .

For instance, comparing Eq. 1 and the results above, we can see that  $y_1 = 2x_1 - (a(\mathbf{x}) + LSB(a(\mathbf{x}))) + w_1$  in Wang *et al.*'s method, while in the proposed method,  $2x_1 - \bar{\mathbf{x}}_{\mathbf{n-1}} - d_e \leq y_1 \leq 2x_1 - \bar{\mathbf{x}}_{\mathbf{n-1}}$ . In Wang *et al.*'s method, since  $a(\mathbf{x})$  is the mean value of  $\mathbf{x}_n$ ,  $y_1$  is converted to  $2x_1 - \bar{\mathbf{x}}_n - LSB(\mathbf{x}_n) + w_1$ . In Alattar's method, as the value of n is increased and all the to-be-embedded-bits are '1', the value of  $d_e$  also becomes larger correspondingly, and is getting more and more closer to  $\lfloor \frac{n+3}{2} \rfloor$ . Therefore, the distortion introduced by Alattar's method becomes more and more higher than that by Wang *et al.*'s method with the value of n increased. To decrease the distortion induced by Alattar's algorithm itself, n = 4 is selected in the proposed method.

3.4. Data Embedding. To prevent the overflow/underflow, each watermarked pixel is classified into one of three sets:  $NO_s$ ,  $\mathbf{x}_S$  and  $\mathbf{x}_C$ . Set  $\mathbf{x}_S$  contains all sub-blocks with smooth  $\mathbf{x_{n-1}}$  whose all (n-1) watermarked values fall in the range of [0, 255], i.e.,  $0 \leq y_i \leq 255(1 \leq i \leq n-1 \text{ and } i \in \mathbb{Z})$  and  $0 \leq y_n \leq 255$ . And meanwhile, set  $\mathbf{x}_C$  contains all sub-blocks with complex  $\mathbf{x_{n-1}}$  whose all (n-1) watermarked values fall in the range of [0, 255] and  $0 \leq y_n \leq 255$ . And meanwhile, set  $\mathbf{x}_C$  contains all sub-blocks with complex  $\mathbf{x_{n-1}}$  whose all (n-1) watermarked values fall in the range of [0, 255] and  $0 \leq y_n \leq 255$ . After any modification above (including Eq. 6 to Eq. 10), if overflow/underflow occurs, i.e.,  $y_i > 255$  or  $y_i < 0(1 \leq i \leq n \text{ and } i \in \mathbb{Z})$ , then this sub-block belongs to set  $NO_s$ .

A location map is generated in which the locations of the sub-blocks belongs to  $NO_s$  are marked by '1' while the others are marked by '0'. The location map is compressed losslessly by an arithmetic encoder and the resulting bitstream is denoted by  $\mathcal{L}$ .  $L_S$  is the bit length of  $\mathcal{L}$ .

For  $\mathbf{x} \in \mathbf{x}_S$ , if any difference value  $d_i (i \in \{1, 2, \dots, n-2\})$  of sub-set  $\mathbf{x}_{n-1}$  falls in the range of  $[-pT_h, pT_h)$ , then it is classified into  $E_2$ , otherwise, it belongs to  $H_2$ . Similarly, for  $\mathbf{x} \in \mathbf{x}_C$ , if  $d_i \in [-pT_h, pT_h)$ , then this  $d_i$  is classified into  $E_1$ . Otherwise, it is classified

into  $H_1$ . For  $x_n$  of sub-blocks belonging to set  $\mathbf{x}_S \bigcup \mathbf{x}_C$ , if its corresponding watermarked value  $y_n$  obtained from Eq. 10 does not suffer from overflow/underflow, and meanwhile,  $d_{n-1} = x_n - \bar{\mathbf{x}}_{n-1}$  falls inside the interval  $[-pT_h, pT_h]$ , then 1-bit watermark is embedded into  $x_n$ . For the convenience of description, this  $x_n$  is grouped into set  $E_3$ , otherwise, it belongs to  $H_3$ . For sub-set  $\mathbf{x}_{n-1}$  belonging to  $E_2$ , each can at most carry 2(n-1) watermark bit, so the maximum hiding capacity is  $C_{ap} = 2||E_2|| + ||E_1|| + ||E_3|| - L_S$  bits, where  $||\cdot||$ represents the cardinality of a set.

For each sub-block, if it is in  $NO_s$ , then **x** is kept unaltered, i.e.,  $\mathbf{y} = \mathbf{x}$ . If  $\mathbf{x}_{n-1}$  belongs to set  $E_2$ , then it is expanded twice according to Eq. (6). If  $\mathbf{x}_{n-1}$  belongs to  $H_2$ , it will be shifted according to Eq. (7). If **x** belongs to set  $E_1$ , then it is expanded only once according to Eq. (8). If  $\mathbf{x}_{n-1}$  belongs to  $H_1$ , it will be shifted according to Eq. (9). If  $x_n$ belongs to set  $E_3 \cup H_3$ , then it is modified according to Eq. (10).

After the first  $L_S$  pixels have been processed, the LSBs of y is appended to the payload  $\mathcal{P}$  and replaced by the compressed location map  $\mathcal{L}$ . After all the sub-blocks are processed, a new marked image  $I_w$  is obtained.

3.5. Data Extraction and Image Restoration. The LSBs of the pixels in  $I_w$  are collected into a bitstream  $\mathcal{B}$  according to the same order as in embedding.  $\mathcal{B}$  is decompressed by an arithmetic decoder to retrieve the location map.

For each watermarked sub-block, if its location is associated with '0' in the location map, then it is ignored. Otherwise, if  $x_n$  is retrieved as following.

$$x_{n} = \begin{cases} \left\lfloor \frac{y_{n} - \mathbf{x}_{n-1}}{2} \right\rfloor + \bar{\mathbf{x}}_{n-1} & d_{n-1}' \in [-2pT_{h}, 2pT_{h} - 1] \\ y_{n} + pT_{h} & d_{n-1}' \leq -2pT_{h} - 1 \\ y_{n} - pT_{h}, & d_{n-1}' \geq 2pT_{h} \end{cases}$$
(12)

where  $d'_{n-1} = y_n - \mathbf{x_{n-1}}$ . Correspondingly, the embedded watermark is extracted by the following formula:  $b_i = y_n - 2\lfloor \frac{y_n - \bar{\mathbf{x}_{n-1}}}{2} \rfloor$ . After  $x_n$  is restored,  $d_{n-1} = x_n - \bar{\mathbf{x}_{n-1}}$  is calculated, if  $|d_{n-1}| \leq vTh$ , then  $x_i$  is retrieved

After  $x_n$  is restored,  $d_{n-1} = x_n - \bar{\mathbf{x}}_{n-1}$  is calculated, if  $|d_{n-1}| \leq vTh$ , then  $x_i$  is retrieved as follows.

$$d_{i} = \begin{cases} \lfloor \frac{d'_{i}}{4} \rfloor & d'_{i} \in [-4pT_{h}, 4pT_{h} - 1] \\ d'_{i} + 3 \times pT_{h} & d'_{i} \le -4pT_{h} - 1 \\ d'_{i} - 3 \times pT_{h}, & d'_{i} \ge 4pT_{h} \end{cases}$$
(13)

where  $d'_i = y_{i+1} - y_i$  and  $i \in \{1, 2, \dots, n-2\}$ . Correspondingly, the embedded watermark is extracted by the following formula:  $e_i = d'_i - 4\lfloor \frac{d'_i}{4} \rfloor$ . Otherwise,  $x_i$  is retrieved as follows.

$$d_{i} = \begin{cases} \lfloor \frac{d'_{i}}{2} \rfloor & d'_{i} \in [-2pT_{h}, 2pT_{h} - 1] \\ d'_{i} + pT_{h} & d \leq -2pT_{h} - 1 \\ d'_{i} - pT_{h}, & d \geq 2pT_{h} \end{cases}$$
(14)

where  $d'_i = y_{i+1} - y_i$  and  $i \in \{1, 2, \dots, n-2\}$ . Correspondingly, the embedded watermark is extracted by the following formula:  $e_i = d'_i - 2\lfloor \frac{d'_i}{2} \rfloor$ . Finally, the retrieved difference valuess and the mean value  $\bar{\mathbf{x}}_{n-1}$  are substituted into Eq. 5 to obtain the original pixel value  $x_i$ .

4. Experimental results. The capacity vs. distortion comparisons among the proposed method, Wang *et al.*'s, Weng's and Alg. D2 are shown in Figs. 2 and 5. Triple embedding is performed in Weng's method and Alg. D2 to achieve the embedding bit-rate of over 1.0 bpp.



FIGURE 2. Capacity vs. Distortion Comparison on 'Lena'

Since Wang *et al.*'s method with  $4 \times 4$  sized block performs the best, the location map only is  $\frac{1}{16}$  size, relative to any original grayscale image. And therefore, small embedding capacity still can be obtained even when the threshold is set to a small integer value. And meanwhile, Wang et al.'s method only modified the embeddable sub-block while making almost no modifications to the other sub-blocks. Therefore, the embedding-induced distortion can be kept at a low level, even if the embedding bit-rate is low. However, with embedding bit-rate increased, the distortion is increasingly severe. In the proposed method, for any smooth block, if its difference value between two neighboring pixels falls into the range of  $[-pT_h, pT_h]$ , then it will be expanded twice. For the other smooth blocks, their pixel values must be shifted so as to ensure reversibility. In a word, the reversibility is achieved with additional costs of distortion. Hence, at the low embedding rates, Wang *et al.*'s method provides better performance than the proposed method. However, the number of smooth blocks used in shifting is decreased as the embedding rate is increased. The proposed method performs well, and significantly outperforms Wang et al.'s method for larger embedding rates. For 'Lena'image, it also can be seen from Fig. 2, relative to Wang et al.'s method and the proposed method, both higher embedding rates and larger PSNR values are obtained by Weng's and Alg. D2 exceeding 0.7 bpp. The proposed method exceeds all methods at about 0.9 bpp.

Baboon' is a typical image with large areas of complex texture, so the obtained bit-rate is slightly lower at the same PSNR. Fig. 3 show that the proposed method achieves higher embedding capacity with lower embedding distortion than the others. The proposed method have the same performance with the other methods.

For 'Barbara' images, it can be seen from Fig. 4 that the proposed method have better performance than Weng's method and Alg. D2 at almost all embedding rates. When the embedding rate is over 0.55 bpp, the proposed method, Weng 's method and Alg. D2 all exceed Wang *et al.*'s method in both embedding rates and PSNR values. As a result, the proposed method can achieve larger embedding rates while keeping higher PSNR in the field of reversible watermarking based on integer transform. There is similar situation for 'Goldhill' image (shown in Fig. 5).

5. Conclusions. A new reversible watermark scheme capable of mostly carrying 2n-3 bits into one *n*-sized image block in a single embedding process have been presented in this paper. With a variant relation between one pixel and the mean value of the other (n-1) pixels, all the sub-blocks are divided into two classes: smooth block and complex block. More bits are embedded into smooth blocks while fewer bits into complex blocks.

Through a series of experiments, the proposed method has better performance than the other methods at larger embedding rates.

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FIGURE 3. Capacity vs. Distortion Comparison on 'Baboon'



FIGURE 4. Capacity vs. Distortion Comparison on 'Barbara'



FIGURE 5. Capacity vs. Distortion Comparison on 'Plane'

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