Journal of Information Hiding and Multimedia Signal Processing ©2014 ISSN 2073-4212
Ubiquitous International Volume **5**, Number **4**, October **2014**

Multi-Channel Quantum Image Representation based on Phase Transform and Elementary Transformations

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Received December, 2013; revised March, 2014

Abstract. *This paper proposed a novel representation for color image on quantum computer and further explored the basic color and geometric transformations on it. First, a multi-channel representation for color quantum image based on phase transform (CQIPT) is introduced in the form of a normalized state which captures information about colors RGB, transparency and positions. Then, a constructive polynomial preparation is introduced for CQIPT. Finally, the elementary transformations on CQIPT such as color channels swapping, one color operations, transparency adjustments and geometric transformations are researched. Analysis shows that the proposed CQIPT is more efficient than other representations for many phase-encoding based quantum image processing.*
Keywords: Quantum computation, Color quantum image, Channel swapping operation, One channel operation, Geometric transformation

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1. **Introduction.** A quantum computer is a physical machine that can accept input states which represent a coherent superposition of many disparate possible inputs and subsequently evolve them into a corresponding superposition of outputs [1]. Any quantum algorithm or unitary operation can be decomposed into a circuit composed by a succession of basic unitary gates that act on one or two qubits [2]. In quantum circuit models of computation, designing circuits is necessary to realize and analyze any quantum algorithm [3]. Physical implementations of the qubits and their corresponding gates have been introduced in the literatures [4].

The natural parallelism of quantum computation makes many classical problems processed as a lower computation complexity. Therefore, quantum image processing has peculiarly been a hot topic in quantum computation and information. However, the first problem to be solved in this field is how to represent image in quantum computer, which is called quantum images. Some methods for representing gray-scale quantum images have been proposed. The quantum image is represented by color, a quantum state detected from monochromatic electromagnetic waves through special machines and position, the storing unit was named Qubit Lattice [5]. Latorre brought out a method that mapped the pixels into the Real Ket of the Hilbert space to complete image compression combined with pixel states [6]. A flexible representation of quantum image (FRQI) [7] encoded the intensity and position of gray image into one quantum state which keeps the classical properties of color and position. An enhanced quantum representation (NEQR) for digital images is proposed in [8], which improves the representation of FRQI. In [9], the authors focus on the quantum image representation using qutrits (3-level quantum systems). Moreover, a quantum image representation for log-polar image (QUALPI) is proposed for the storage and processing of images sampled in log-polar coordinates [10]. All the above representations are serviced for gray scale image. However, color is one of the most important ways to represent images because most objects in nature are colorful. Therefore, based on FRQI, a multi-channel representation for quantum images (MCRQI) using RGBα color space is proposed in [11] . MCRQI is represented and prepared based on rotation gates and is flexible to realize some classic-like operations. However, many other quantum gates such as phase gate are usually utilized in quantum algorithms [12], rotation gate based representation, MCRQI, isn't flexible enough when processed in this condition.

Based on the fact that the images in the real life are usually colorful, color quantum images representation is explored in this paper. Disparate from the representation MCRQI in [11], the CQIPT proposed in this paper is prepared mainly by controlled phase gates. Same as MCRQI, the CQIPT is easily processed for color transformations, transparency adjustments and geometric transformations. Moreover, it is especially flexible for many image processing and security algorithms based on phase encoding.

The rest of the paper is organized as follows. Section 2 gives the pattern of CQIPT and the polynomial preparation theories are studied in Section 3. Furthermore, Section 4 researches the transformations about disparate channels , single channel and positions. Analysis acting upon CQIPT is proposed in Section 5. Finally, Section 6 concludes the paper.

2. **Color quantum image based on phase transform (CQIPT).** To simplify the problem, we suppose that the size of a color image is $2^n \times 2^n$, the concrete pattern of CQIPT is shown in Eq. (1),

$$|I(\theta)\rangle = \frac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} \left|c^j_{RGB\alpha}\right\rangle \otimes |j\rangle\,, \tag{1}$$

here, different from MCRQI using rotation gate in [11], the color encoding $\left|c^j_{RGB\alpha}\right\rangle$ carrying the information R, G, B and transparency α is defined as,

$$\begin{aligned}\left|c^j_{RGB\alpha}\right\rangle = {} & |000\rangle + e^{i\theta_{Rj}}\,|001\rangle + |010\rangle + e^{i\theta_{Gj}}\,|011\rangle + \\ & + |100\rangle + e^{i\theta_{Bj}}\,|101\rangle + |110\rangle + e^{i\theta_{\alpha j}}\,|111\rangle\,,\end{aligned} \tag{2}$$

wherein, $i = \sqrt{-1}$, $\theta_{Xj} \in \left[0, \frac{\pi}{2}\right]$, $X \in \{R, G, B, \alpha\}$, $j = 0, 1, ..., 2^{2n} - 1$ encodes the color information and $|j\rangle$ encodes the corresponding position of the quantum images. The position information includes two parts: the vertical and horizontal coordinates. Considering a color quantum image in $2n$-qubits system,

$$|j\rangle = |y\rangle\,|x\rangle = |y_{n-1}y_{n-2}\cdots y_0\rangle\,|x_{n-1}x_{n-2}\cdots x_0\rangle\,, \tag{3}$$

$$x, y \in \{0, 1, ..., 2^n - 1\},$$

$$|y_i\rangle\,, |x_i\rangle \in \{|0\rangle\,, |1\rangle\}, i = 0, 1, \cdots, n-1,$$

where, $|y\rangle$ encodes the first n-qubits along the vertical location and $|x\rangle$ encodes the second n-qubits along the horizontal axis. The CQIPT state is a normalized state, i.e.

$$
\begin{aligned}
\| |I(\theta)\rangle \| &= \frac{1}{2^{n+3/2}} \sqrt{\sum_{j=0}^{2^{2n}-1} (1 + |e^{i\theta_{Rj}}|^2 + 1 + |e^{i\theta_{Gj}}|^2 + 1 + |e^{i\theta_{Bj}}|^2 + 1 + |e^{i\theta_{\alpha j}}|^2)} \\
&= \frac{1}{2^{n+3/2}} \sqrt{2^{2n} \cdot 2^3} = 1.
\end{aligned} \tag{4}
$$

CQIPT can be easily generalized to represent any color image of size $2^m \times 2^n$.

3. **Polynomial preparation for CQIPT.** Generally, the initialized state of quantum computation is supposed to be $|0\rangle^{\otimes 2n+3}$, the aim of preparation is to transform the initialized state to the desired state $|I(\theta)\rangle$ using unitary transforms described by unitary matrices.

Lemma 3.1. *Given angle vectors $\theta_X = (\theta_{X0}, \theta_{X1}, \cdots, \theta_{X(2^{2n}-1)})$,$X \in \{R, G, B, \alpha\}$, there is a unitary transformation P that turns the quantum computers from initialized state $|0\rangle^{\otimes 2n+3}$to desired CQIPT state $|I(\theta)\rangle$, composed by Hadamard and controlled phase transformations.*

Proof: The preparation process can be divided into two steps A and B. Suppose two single qubit gate, identity transform and Hadamard gate,

$$
I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.
$$

The first transform is applying H gate on every qubit of $|0\rangle^{\otimes 2n+3}$, i.e., $A = H^{\otimes 2} \otimes H \otimes H^{\otimes 2n}$ on $|0\rangle^{\otimes 2n+3} = |0\rangle^{\otimes 2} \otimes |0\rangle \otimes |0\rangle^{\otimes 2n}$, we have

$$
\begin{aligned}
A(|0\rangle^{\otimes 2n+3}) &= \tfrac{1}{2^{n+3/2}} \sum_{l=0}^{3} |l\rangle \otimes (|0\rangle + |1\rangle) \otimes \sum_{j=0}^{2^{2n}-1} |j\rangle \\
&= |W\rangle .
\end{aligned} \tag{5}
$$

Then, we construct four 8×8controlled phase matricesP_{Ri},P_{Gi},$P_{Bi}$and$P_{\alpha i}$ as follows,

$$
P_{Xi} = \left(\sum_{j=0, j \neq f(X)}^{3} |j\rangle \langle j| \otimes I \right) + |f(X)\rangle \langle f(X)| \otimes P(\theta_{Xi}),
$$

wherein, $P(\theta_{Xj})$ is phase transform and $P(\theta_{Xj}) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta_{Xj}} \end{pmatrix}$, $X \in \{R, G, B, \alpha\}$,

$$
f(X) = \begin{cases} 0, & X = R, \\ 1, & X = G, \\ 2, & X = B, \\ 3, & X = \alpha. \end{cases} \tag{6}
$$

Thus, we obtain $P'_i = P_{Ri} P_{Gi} P_{Bi} P_{\alpha i}$, $i = 0, 1, ..., 2^{2n} - 1$ and furthermore, construct transform

$$
P_i = I^{\otimes 3} \otimes \sum_{j=0, j \neq i}^{2^{2n}-1} |j\rangle \langle j| + P'_i \otimes |i\rangle \langle i| . \tag{7}
$$

It is easily proved that P_i is a unitary matrix, thus, applying P_k on $|I(\theta)\rangle$ gives us

$$P_k(|W\rangle) = \frac{1}{2^{n+3/2}}\left\{\left[\left(I^{\otimes 2}\left(\sum_{l=0}^{3}|l\rangle\right)\right)\otimes I\left(|0\rangle+|1\rangle\right)\right]\otimes\left[\left(\sum_{j=0,j\neq k}^{2^{2n}-1}|j\rangle\langle j|\right)\left(\sum_{i=0}^{2^{2n}-1}|i\rangle\right)\right]+\right.$$
$$\left.+\left[P'_k\left(\sum_{l=0}^{3}|l\rangle\otimes(|0\rangle+|1\rangle)\right)\right]\otimes\left[(|k\rangle\langle k|)\left(\sum_{i=0}^{2^{2n}-1}|i\rangle\right)\right]\right\}$$
$$= \frac{1}{2^{n+3/2}}\left[\sum_{l=0}^{3}|l\rangle\otimes(|0\rangle+|1\rangle)\otimes\sum_{j=0,j\neq k}^{2^{2n}-1}|j\rangle+\left|c^k_{RGB\alpha}\right\rangle\otimes|k\rangle\right], \tag{8}$$

$$P_mP_k(|W\rangle) = P_m\left(P_k|W\rangle\right)$$
$$= \frac{1}{2^{n+3/2}}\left[\sum_{l=0}^{3}|l\rangle\otimes(|0\rangle+|1\rangle)\otimes\sum_{j=0,j\neq k,m}^{2^{2n}-1}|j\rangle+\left|c^k_{RGB\alpha}\right\rangle\otimes|k\rangle+\left|c^m_{RGB\alpha}\right\rangle\otimes|m\rangle\right]. \tag{9}$$

From (9), it is clear that

$$B(|W\rangle) = \left(\prod_{i=0}^{2^{2n}-1}P_i\right)|W\rangle = |I(\theta)\rangle. \tag{10}$$

Therefore, unitary transform $P = BA$ can turn quantum computer from initialized state to the CQIPT.

Then, we focus on the feasibility of broken down the transform P into simple gates such as NOT gate, Hadamard, and CNOT gates. The circuit of NOT and CNOT gate are illuminated in Figs. 1 and 2.

Figure 1. NOT gate

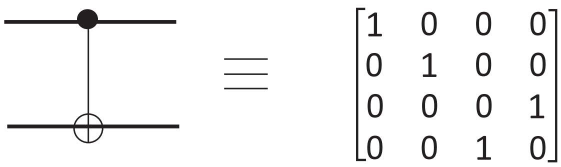


Figure 2. CNOT gate

Corollary 3.1. *The unitary transform P in Lemma 1 can be implemented by Hadamard gate, CNOT and controlled phase gate $C^{2n+2}\left(P\left(\frac{\theta_{X_i}}{2^{2n+1}}\right)\right)$, $i = 0, 1, ..., 2^{2n}-1$.*

Proof: From the proof of Lemma 1, the transform P is composed of BA. The transform A can be directly realized by $2n+3$ Hadamard gates, and the transform B is constructed by $\prod_{i=0}^{2^{2n}-1}P_i$, where

$$P_i = I^{\otimes 3}\otimes\sum_{j=0,j\neq i}^{2^{2n}-1}|j\rangle\langle j| + P'_i\otimes|i\rangle\langle i|,$$

$$P_i' = P_{Ri}P_{Gi}P_{Bi}P_{\alpha i},$$

$$P_{Xi} = C^2\left(P\left(\theta_{Xi}\right)\right).$$

Thus, P_i can be implemented by $C^{2n+2}\left(P\left(\theta_{Xi}\right)\right)$ and NOT gate [13]. Furthermore, the result in [3] implies that $C^{2n+2}\left(P\left(\theta_{Xi}\right)\right)$ can be broken down into $2^{2n+2}-1$simple gates, $P\left(\frac{\theta_{Xi}}{2^{2n+1}}\right)$, and $2^{2n+2}-2$CNOT gates.

The total number of simple operations used to prepare CQIPT is

$$\begin{aligned} & 2n+3+4\times 2^{2n}\times(2^{2(n+1)}-1+2^{2(n+1)}-2) \\ = & \, 2\times(4\times 2^{2n})^2-3\times(4\times 2^{2n})+2n+3. \end{aligned}$$

This is quadratic to the total4×2^{2n}angle values.

Theorem 3.1. *Given 4 vectors $\theta_X = (\theta_{X0}, \theta_{X1}, \cdots, \theta_{X(2^{2n}-1)}), X \in \{R, G, B, \alpha\}$, there is a unitary transform P that turns quantum computer from the initial state $|0\rangle^{\otimes 2n+3}$ to the CQIPT state,$|I(\theta)\rangle = \frac{1}{2^{n+3/2}}\sum\limits_{j=0}^{2^{2n}-1}\left|c^j_{RGB\alpha}\right\rangle \otimes |j\rangle$, composed of polynomial number of simple gate.*

Proof: Coming from Lemma 3.1 and Corollary 3.1.

4. **Elementary transformations for CQIPT.** For classical images, kinds of image processing operations such as geometric transformations, color transformations and image compression etc. have been studied deeply. But as for quantum images, corresponding processing is still in his infancy. Based on FRQI, strategies and fast geometric transformations are proposed in [14][15] and efficient color transformations are proposed in [2]. Based on MCRQI, channel swapping operation and one channel operation is proposed in [11]. Actually, for disparate representations for quantum images, image processing circuits on them should be designed specially. Therefore, in this section, some elementary transformations about channel swapping, one channel swapping and transparency transformations for CQIPT are researched, respectively.

4.1. **Channel swapping operation.** Channel Swapping Operation (CSO) on MCRQI was defined in [11], similarly, three sorts of CSO on CQIPT, T_{RG}, $T_{GB}$and T_{RB} swapping the values between R and G, G and B or R and B are designed in this section.

1. Swapping between R and G channel

To realize the swapping operation between R and G channel, the desired state is as follows,

$$\begin{aligned} T_{RG}(|I(\theta)\rangle) &= \frac{1}{2^{n+3/2}}\sum_{j=0}^{2^{2n}-1} T_{RG}\left(\left|c^j_{RGB\alpha}\right\rangle\right)\otimes|j\rangle \\ &= \frac{1}{2^{n+3/2}}\sum_{j=0}^{2^{2n}-1}\left|c^j_{GRB\alpha}\right\rangle\otimes|j\rangle, \end{aligned} \tag{11}$$

where,

$$\begin{aligned} \left|c^j_{GRB\alpha}\right\rangle = & |000\rangle + e^{i\theta_{Gj}}|001\rangle + |010\rangle + e^{i\theta_{Rj}}|011\rangle + \\ & + |100\rangle + e^{i\theta_{Bj}}|101\rangle + |110\rangle + e^{i\theta_{\alpha j}}|111\rangle. \end{aligned}$$

The quantum circuit to realize this operation T_{RG} is illustrated in Fig.3.

The zero controlled NOT gate is shown in Fig.4

2. Swapping between G and B channel

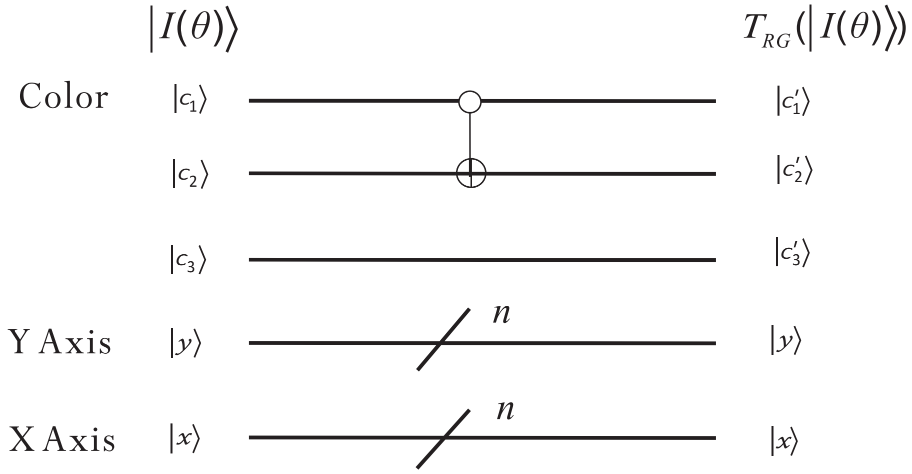


Figure 3. Quantum circuit for swapping between channels R and G

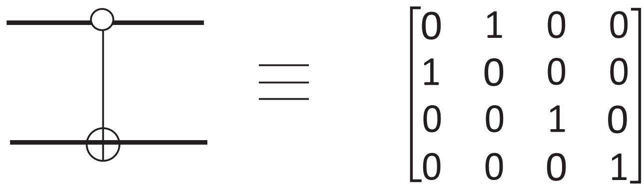


Figure 4. Zero controlled NOT gate

Similar as T_{RG}, the swapping operation and the corresponding quantum circuit between G and B channel is shown in Eq.(12) and Fig. 5, separately,

$$\begin{aligned} T_{GB}(|I(\theta)\rangle) &= \frac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} T_{GB}\left(\left|c^j_{RGB\alpha}\right\rangle\right) \otimes |j\rangle \\ &= \frac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} \left|c^j_{RBG\alpha}\right\rangle \otimes |j\rangle, \end{aligned} \tag{12}$$

where,

$$\begin{aligned} \left|c^j_{RBG\alpha}\right\rangle = |000\rangle + e^{i\theta_{Rj}}|001\rangle + |010\rangle + e^{i\theta_{Bj}}|011\rangle + \\ + |100\rangle + e^{i\theta_{Gj}}|101\rangle + |110\rangle + e^{i\theta_{\alpha j}}|111\rangle. \end{aligned}$$

And the controlled swapping gate is shown in Fig.6.

3. Swapping between R and B channel

Moreover, the swapping between R and B channel can be realized by Eq. (13) and Fig. 7.

$$\begin{aligned} T_{RB}(|I(\theta)\rangle) &= \frac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} T_{RB}\left(\left|c^j_{RGB\alpha}\right\rangle\right) \otimes |j\rangle \\ &= \frac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} \left|c^j_{BGR\alpha}\right\rangle \otimes |j\rangle, \end{aligned} \tag{13}$$

where,

$$\begin{aligned} \left|c^j_{BGR\alpha}\right\rangle = |000\rangle + e^{i\theta_{Bj}}|001\rangle + |010\rangle + e^{i\theta_{Gj}}|011\rangle + \\ + |100\rangle + e^{i\theta_{Rj}}|101\rangle + |110\rangle + e^{i\theta_{\alpha j}}|111\rangle. \end{aligned}$$

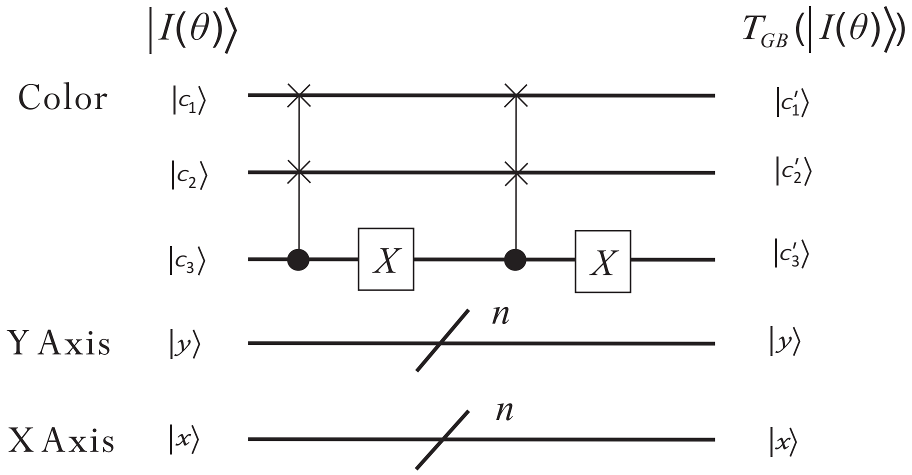
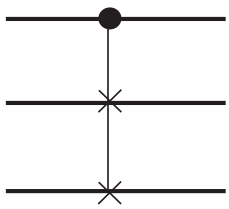


FIGURE 5. Quantum circuit for swapping between channels G and B



$$\equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

FIGURE 6. Controlled swapping gate

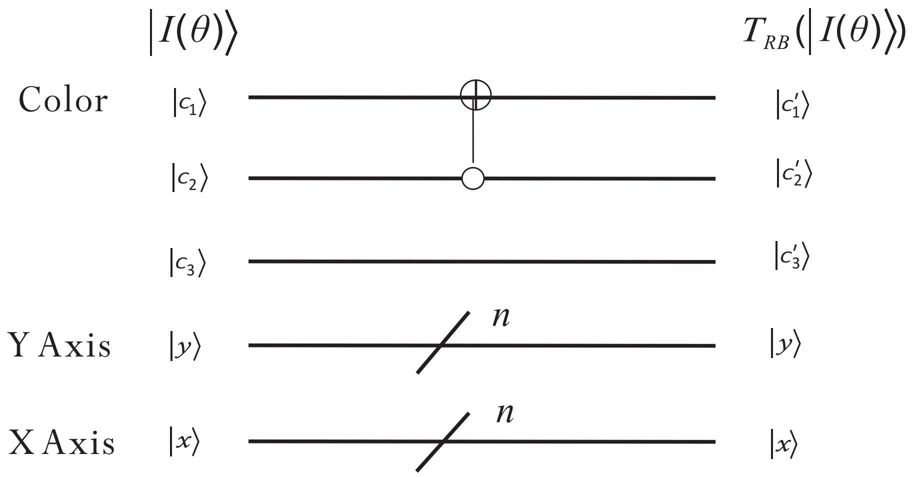


FIGURE 7. Quantum circuit for swapping between channels R and B

4.2. **one channel operation.** As for only one color operation, because all the color and position information is entangled together, it is impossible to distinguish the pixels with different color scales using basic quantum gates and operate them individually. But changing the same phase angel of one channel to all positions is easily realized. Fig.8

shows the circuit to realize R channel from $|000\rangle + e^{i\theta_{Rj}}|001\rangle$to $|000\rangle + e^{i(\theta_{Rj}+\phi_R)}|001\rangle$ for all $j = 0, 1, ..., 2^{2n} - 1$, where the transform matrix is

$$\begin{aligned} C_R(|I(\theta)\rangle) &= \tfrac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} C_R\left(\left|c^j_{RGB\alpha}\right\rangle\right) \otimes |j\rangle \\ &= \tfrac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} \left|c^j_{R'GB\alpha}\right\rangle \otimes |j\rangle , \end{aligned} \tag{14}$$

where,

$$\begin{aligned} \left|c^j_{R'GB\alpha}\right\rangle = |000\rangle &+ e^{i(\theta_{Rj}+\phi_R)}|001\rangle + |010\rangle + e^{i\theta_{Gj}}|011\rangle + \\ &+ |100\rangle + e^{i\theta_{Bj}}|101\rangle + |110\rangle + e^{i\theta_{\alpha j}}|111\rangle . \end{aligned}$$

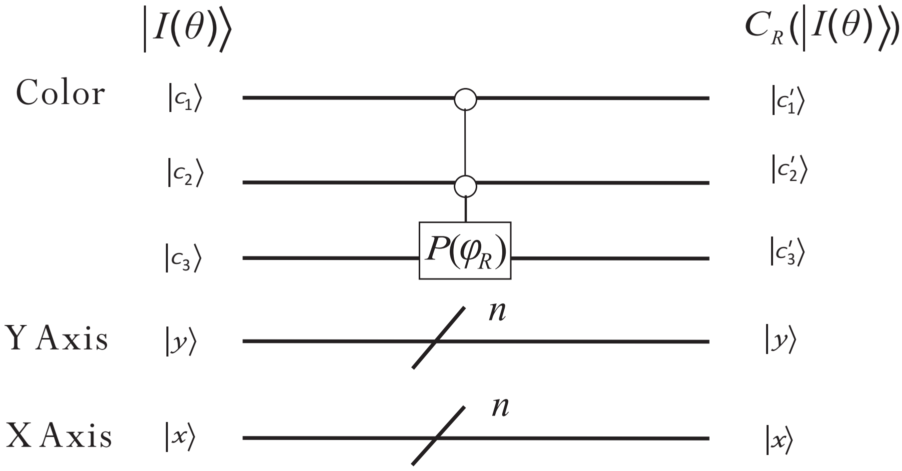


FIGURE 8. Quantum circuit for changing R channel by angle φ_R

Similarly, the operations and circuits to realize G, B and α channel changing angles φ_G, φ_B and φ_α are shown in Eqs. (15)-(17) and Figs. 9-11.

$$\begin{aligned} C_G(|I(\theta)\rangle) &= \tfrac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} C_G\left(\left|c^j_{RGB\alpha}\right\rangle\right) \otimes |j\rangle \\ &= \tfrac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} \left|c^j_{RG'B\alpha}\right\rangle \otimes |j\rangle , \end{aligned} \tag{15}$$

here,

$$\begin{aligned} \left|c^j_{RG'B\alpha}\right\rangle = |000\rangle &+ e^{i\theta_{Rj}}|001\rangle + |010\rangle + e^{i(\theta_{Gj}+\phi_G)}|011\rangle + \\ &+ |100\rangle + e^{i\theta_{Bj}}|101\rangle + |110\rangle + e^{i\theta_{\alpha j}}|111\rangle . \end{aligned}$$

And

$$\begin{aligned} C_B(|I(\theta)\rangle) &= \tfrac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} C_B\left(\left|c^j_{RGB\alpha}\right\rangle\right) \otimes |j\rangle \\ &= \tfrac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} \left|c^j_{RGB'\alpha}\right\rangle \otimes |j\rangle , \end{aligned} \tag{16}$$

where,

$$\begin{aligned} \left|c^j_{RGB'\alpha}\right\rangle = |000\rangle &+ e^{i\theta_{Rj}}|001\rangle + |010\rangle + e^{i\theta_{Gj}}|011\rangle + \\ &+ |100\rangle + e^{i(\theta_{Bj}+\phi_B)}|101\rangle + |110\rangle + e^{i\theta_{\alpha j}}|111\rangle . \end{aligned}$$

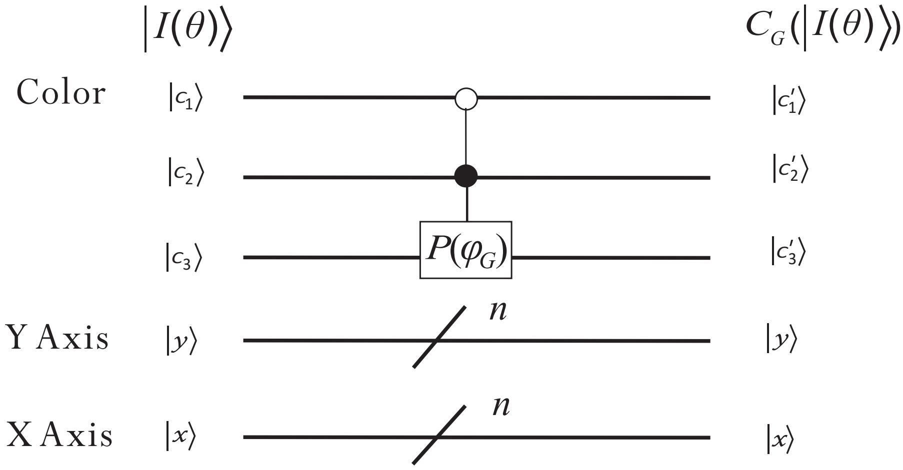


FIGURE 9. Quantum circuit for changing R channel by angle φ_G

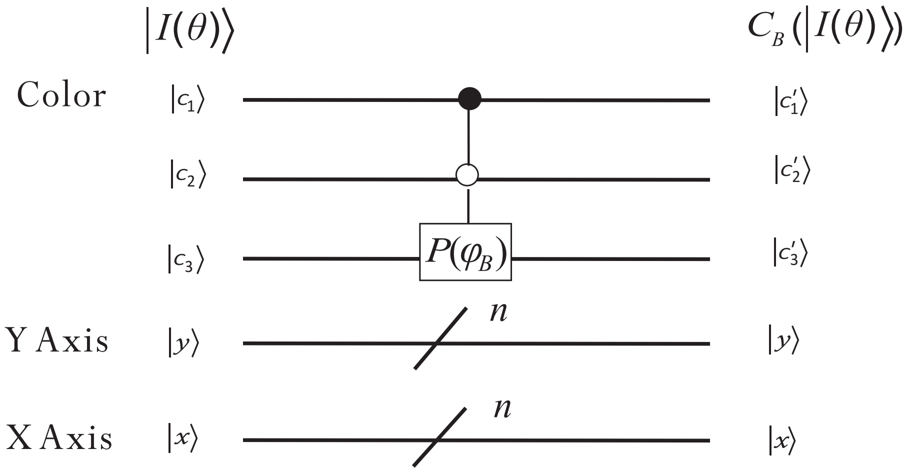


FIGURE 10. Quantum circuit for changing R channel by angle φ_B

Finally,

$$\begin{aligned} C_\alpha(|I(\theta)\rangle) &= \tfrac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} C_\alpha\left(\left|c^j_{RGB\alpha}\right\rangle\right) \otimes |j\rangle \\ &= \tfrac{1}{2^{n+3/2}} \sum_{j=0}^{2^{2n}-1} \left|c^j_{RGB\alpha'}\right\rangle \otimes |j\rangle\,, \end{aligned} \tag{17}$$

here,

$$\begin{aligned} \left|c^j_{RGB\alpha'}\right\rangle = |000\rangle + e^{i\theta_{Rj}}|001\rangle + |010\rangle + e^{i\theta_{Gj}}|011\rangle + \\ + |100\rangle + e^{i\theta_{Bj}}|101\rangle + |110\rangle + e^{i(\theta_{\alpha j}+\phi_\alpha)}|111\rangle\,. \end{aligned}$$

If just one value of some channel belongs to some special position swaps, the above circuits are not applicable for it any more. A feasible transform to realize the aim of changing only one value on some position and keep the others unchangeable is constructed as follows. Suppose we just want to change X channel ($X \in \{R, G, B, \alpha\}$) on the position $|i\rangle$ from θ_{Xi} to ψ_{Xi}, we can design transform,

$$C_{Xi} = I^{\otimes 3} \otimes \sum_{j=0, j\neq i}^{2^{2n}-1} |j\rangle\langle j| + C'_{Xi} \otimes |i\rangle\langle i|\,, \tag{18}$$

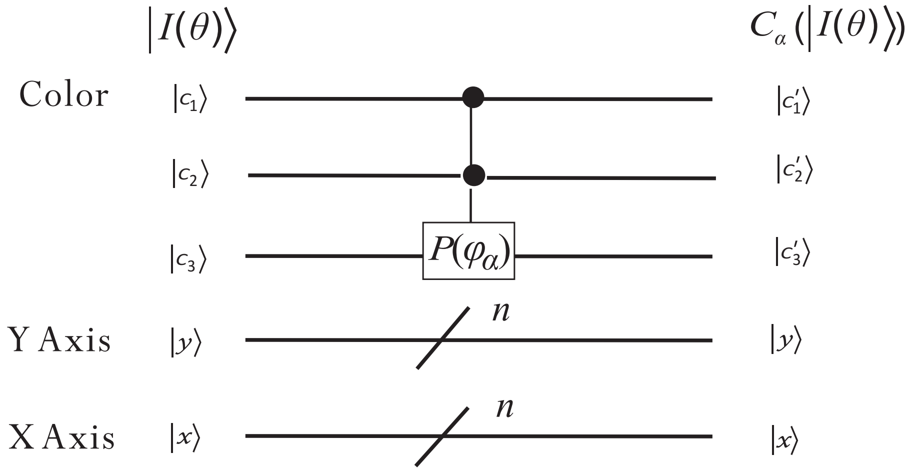


FIGURE 11. Quantum circuit for changing α channel by angle φ_α

$$\begin{aligned} C'_{Xi} =& \left(\sum_{j=0}^{3} |j\rangle\langle j| \otimes I\right)^{f(X)} \left[\sum_{j=0, j\neq f(X)}^{3} |j\rangle\langle j| \otimes I + |f(X)\rangle\langle f(X)| \otimes P(\varphi_{Xi}) \right] \\ & \left(\sum_{j=0}^{3} |j\rangle\langle j| \otimes I\right)^{3-f(X)}, \end{aligned} \tag{19}$$

Here, $X \in \{R, G, B, \alpha\}$, $f(X)$ is same as Eq. (6).

4.3. **Geometric transformations.** As for geometric transformations such as position shifting [14], two-point swapping, flip, coordinate swapping and orthogonal rotations [15], the operations are isolated to the color qubits, therefore, the general circuit of above operations can be applied on the latter $2n$-qubits about pixel positions of CQIPT. The general quantum circuit is shown in Fig. 12.

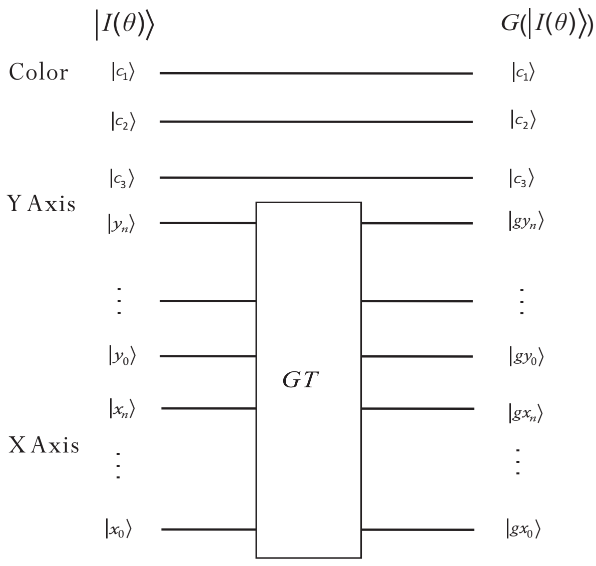


FIGURE 12. General circuit design for geometric transformations on CQIPT

5. **Analysis.** Comparing with the existed multi-channel representation for color image MCRQI in [11], the proposed CQIPT not only has the same preparation complexity and the same flexibility for the elementary transformations, but also have more flexibility for processing on image using phase transformations. Double random-phase encoding (DRPE) technique proposed in [16] combining appropriate transformed domains is widely used in image security such as encryption [17] and information hiding [18]. A typical DRPE technique based image encryption is

$$c(x,y) = FT^{-1}\{FT\{f(x,y)\exp[j2\pi n(x,y)]\}\exp[j2\pi b(\xi,\eta)]\}, \tag{20}$$

$$f(x,y) = FT^{-1}\{FT\{c(x,y)\exp[-j2\pi b(\xi,\eta)]\}\exp[-j2\pi n(x,y)]\}. \tag{21}$$

Where, $f(x,y)$is the plain image, $c(x,y)$is the cipher image, $n(x,y)$and $b(\xi,\eta)$are the two random-phase functions in spatial domain and frequency domain, respectively, which are uniformly distributed in [0, 1]. FT and FT^{-1} represent the Fourier transform and its inverse Fourier transform, respectively. Using the proposed CQIPT and transformations shown in Eqs. (18) and (19), it is more easily than MCRQI [11] to realize the processing such as Eqs. (20) and (21).

6. **Conclusions.** For some special applications of quantum images, especially phase encoding based methods, this paper proposed a novel multi-channel quantum image representation approach CQIPT. Moreover, the elementary transformations about channel swapping operations, one channel swapping operations and general circuits for designing geometric transformations are researched. The corresponding transformations matrices and quantum circuits are designed and they can be easily implemented by basic quantum gates.

As for future work, the results in this paper will be extended to representation for video, the quantum image encryption and quantum image watermarking for publication protection.

Acknowledgment. This work is supported by the National Natural Science Foundation of China (61301099, 61100187, 11201100), Heilongjiang Province Educational Department Funds of China (12521107), Heilongjiang Province Scientific and technical Department Fund of China (A201213) and the Youth Foundation at the Harbin University of Science and Technology (2011YF009).

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